4th Meeting of WPEC Subgroup 33 on
Methods and issues for the combined use of integral experiments and covariance data

November 29-30, 2010
NEA Headquarter, Issy-les-Moulineaux, France

Proposal for How to Determine Error Matrix of Analytical Modeling

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Discussion Points

- **Making the basic agreement to include the analytical modeling error**

  → **If agreed, how to determine them?**

- **(Case 1) Continuous-energy Monte Carlo method**

- **(Case 2) Most-detailed Deterministic method**

- **(Case 3) Combined method**

Confirmation to Include Analytical Modeling Error (*in Principle*)

- Based on the Bayes theorem, i.e., the conditional probability estimation method
  → To maximize the posterior probability that a cross-section set, ***T***, is true, under the condition that the information of integral experiment, ***Re***, is obtained.

$$J(T) = (T-T_0)^t M^{-1}(T-T_0) + [Re-Rc(T)]^t[Ve+Vm]^{-1}[Re-Rc(T)]$$

Minimize the function $J(T)$. → $dJ(T)/dT = 0$

- The adjusted cross-section set ***T'***, and its uncertainty (covariance), ***M'***

$$T' = T_0 + MG^t[GMG^t+Ve+Vm]^{-1}[Re-Rc(T_0)]$$

$$M' = M - MG^t[GMG^t+Ve+Vm]^{-1}GM$$

✓We need ***Ve*** as matrix form,

and, ***Vm*** as matrix form.

- Prediction error induced by the cross-section errors

Before adjustment : GMG^t After adjustment : $GM'G^t$

Where, T_0 : Cross-section set before adjustment
M : Covariance before adjustment
Re : Measured values of integral experiments
Rc : Analytical values of integral experiments
Ve : Experimental errors of integral experiments
Vm : Analytical modeling errors of integral experiments
G : Sensitivity coefficients, $(dR/R)/(d\sigma/\sigma)$

(Case 1)
Continuous-energy Monte Carlo Method

- **Standard deviation: Statistical error value** evaluated by the MC code. (**Confirmation:** To use the Monte Carlo-based C/E values in the adjustment, and to apply the cross-section changes to the 33-group constants imply "the reactor design values obtained by the 33-group constants **must be always corrected** by the corresponding MC calculation." -> related to Case 3.)

JAEA recommends to multiply the one-sigma value by the MC code with ***Factor 2***. -> see next page.

- **Correlation factors: Zero**, even between two reaction rates from one MC run, since one reaction rate is composed of many independent energy-wise contributions.

Abstract—*Biases in the estimators of the variance and intercycle covariances in Monte Carlo eigenvalue calculations are analyzed. The relations among the "real" and "apparent" values of variances and intercycle covariances are derived, where real refers to a true value that is calculated from independently repeated Monte Carlo runs and apparent refers to the expected value of estimates from a single Monte Carlo run. Next, iterative methods based on the foregoing relations are proposed to estimate the standard deviation of the eigenvalue. The methods work well for the cases in which the ratios of the real to apparent values of variances are between 1.4 and 3.1. Even in the case where the foregoing ratio is >5, >70% of the standard deviation estimates fall within 40% from the true value.*

(Ref.) T.Ueki and T.Mori, et al: NSE 125, pp.1-11 (1997)

TABLE II

Relative Errors of the SDs for the Assembly Powers for the Small-Sized SMART Core

Assembly Index	Relative Power	Real SD (%)	Relative Error (%) of Estimated SD			
			Sample SD	Batch Method	Ueki's Method	New Method
1	1.565	3.04	−73.8	−30.5	−11.7	−13.6
2	1.510	2.13	−80.9	−32.3	−12.7	−10.1
3	1.374	1.55	−73.2	−32.6	−14.3	−8.6
4	1.316	1.08	−59.3	−13.6	5.4	−19.9
5	1.236	0.69	−55.9	−9.0	3.4	−19.0
6	1.092	1.29	−63.1	−32.0	−27.8	−20.3
7	1.079	1.36	−64.5	−7.3	15.3	−15.3
8	0.970	1.10	−67.4	−23.4	−4.0	−15.6
9	0.796	1.56	−74.0	−30.2	−14.6	−10.9
10	0.587	2.24	−70.4	−12.0	14.6	−9.4
11	0.445	1.78	−71.1	−31.5	−16.0	−5.5
Average of | relative error | (%)			68.5	23.1	12.7	13.5

(Ref.) H.J.Shim and C.H.Kim: NSE 162, pp.98-108 (2009)

(Case 2)
Most-detailed Deterministic Method

- **Standard deviation: Summation of a certain percent of the correction values** from the basic analytical model to the most-detailed one (i.e., ***estimation from the sensitivity to approximation degrees of the models***).

  ➡ JAEA recommends to take 30%. -> see next page.

- **Correlation factors: Ratio of the common error to total error.** (**Note:** It is the same idea with those of experimental errors.)

Total error of Data A and B:

$$\sigma_{Total,A} = \pm\sqrt{\sigma^2_{Independent,A} + \sigma^2_{Common,A}},\quad \sigma_{Total,B} = \pm\sqrt{\sigma^2_{Independent,B} + \sigma^2_{Common,B}}$$

Correlation factor between Data A and B :

$$\rho_{A,B} = \frac{\sigma_{Common,A} \times \sigma_{Common,B}}{\sigma_{Total,A} \times \sigma_{Total,B}}$$

Comparison of Experimental and Deterministic-based Analytical Modeling Errors (: Case of ADJ2000R)

- Experimental error
  - Follows the evaluation by experimenters like ANL.
- Analytical modeling error
  - Assumes it is proportional to the sensitivity against the degree of modeling detail,
  - Absolute value was decided to make the ratio of the chi-square value to the freedom approx. unity.
- Elimination of abnormal data
  - Excludes if the deviation of C/E value from unity is three times larger than the total uncertainty value.

（confidence level : 1σ）

Core Parameter		Experimental error	Analytical Modeling error
Criti-cality	JUPITER, FCA, etc.	0.04%	**0.17%**
	Los Alamos	**0.1～0.18%**	**0.15%**
F28/F49 Ratio		**2.5%**	1.1%
F25/F49、C28/F49 Ratio		**2.2%**	0.55%
F49 Distribution		**1.0%**	**0.6～1.2%**
Control Rod Worth		**1.2%**	**1.3%**
Sodium Void Reactivity		2%	**5.5～8.8%**
Doppler Reactivity		2.0～3.0%	**5.0～6.6%**

(Ref.) Workshop on Covariances, BNL, Nuclear Data Sheets 109, pp.2778-2784 (2008)

Rule 1: One analytical modeling error component is "proportional" to the correction value (30% here).
Rule 2: Smaller error value between two data is assumed as "common error".
Rule 3: If the sign of correction values between data is opposite, they are assumed as "independent".

Example 1: Weak Correlation Case

Experimental core		ZPPR-9 (: A)			JOYO Mk-I (: B)			Correlation factor
Error classification			Independent	Common		Independent	Common	
keff by basic method		0.99372			0.98060			
Correction by detailed model (unit: pcm)	Transport theory	+248	0	±74	+1760	±523	±74	
	Mesh-size effect	-93	0	±28	-210	±56	±28	
	Ultra-fine energy effect	+103	±31	0	-50	±15	0	
	Multi-drawer effect	+47	±14	0	0	0	0	
	Cell-asymmetry effect	-52	±16	0	0	0	0	
Total		0.99625	±37	±79	0.99560	±526	±79	
			±88			±532		0.14

multiplied by 0.3

assumed as common

assumed as independent

$$\rho_{A,B} = \frac{\sigma_{Common,A} \times \sigma_{Common,B}}{\sigma_{Total,A} \times \sigma_{Total,B}}$$

(Ref.) NEA IRPhE Handbook, DVD (2010)

Example 2: Strong Correlation Case

Experimental core		ZPPR-10A (: A) (600 MWe-class FBR)			ZPPR-10C (: B) (800 MWe-class FBR)			Correlation factor
Error classification			Independent	Common		Independent	Common	
keff by basic method		0.9913			0.9916			
Correction by detailed model (unit: pcm)	Transport theory	+530	±93	±129	+430	0	±129	
	Mesh-size effect	-130	±21	±33	-110	0	±33	
	Ultra-fine energy effect	+150	±16	±42	+140	0	±42	
	Multi-drawer effect	+40	0	±12	+40	0	±12	
	Cell-asymmetry effect	-60	±10	±15	-50	0	±15	
Total		0.9966	±97	±141	0.9961	0	±141	
			±171			±141		**0.82**

$$\rho_{A,B} = \frac{\sigma_{Common,A} \times \sigma_{Common,B}}{\sigma_{Total,A} \times \sigma_{Total,B}}$$

(Ref.) NEA IRPhE Handbook, DVD (2010)

(Case 3)

Combined Method of Deterministic and Monte Carlo Calculation

Needs: *It is practically impossible to always apply the MC calculation <u>to the complicated situation of the design core and the experimental cores</u>, which have detailed power distributions, burnup compositions and reactivity maps, and others.* -> see next page.

$R_{Combined}$(best) = R_{Det}(simplified geom, as-built comp) + {R_{MC}(as-built geom, simplified comp) - R_{Det}(simplified geom, simplified comp)}

- **Standard deviation: Same with (Case 1) MC method**, ***<u>under the condition</u>*** that the correction values are obtained ***through well-organized simulation models*** from the analytical modeling error viewpoint.
- **Correlation factors: Same with (Case 1) MC method.**

An example difficult to evaluate even by MC calculation

- Reaction rate measurement by foils -

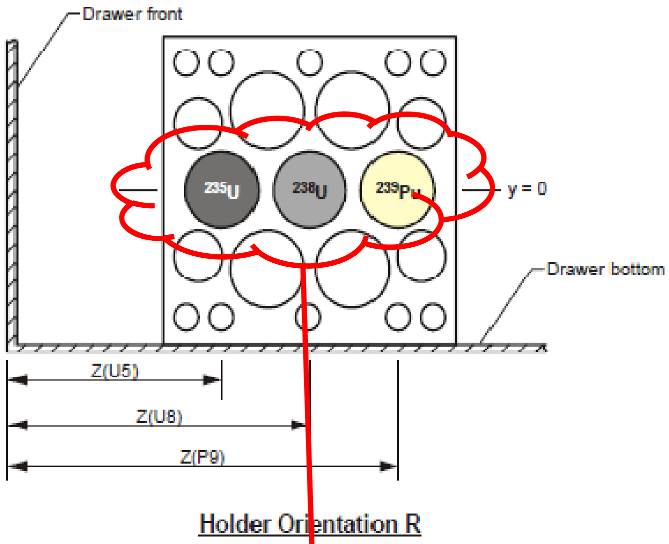


Figure 1.34. Foil Locations in a Drawer (view from side).

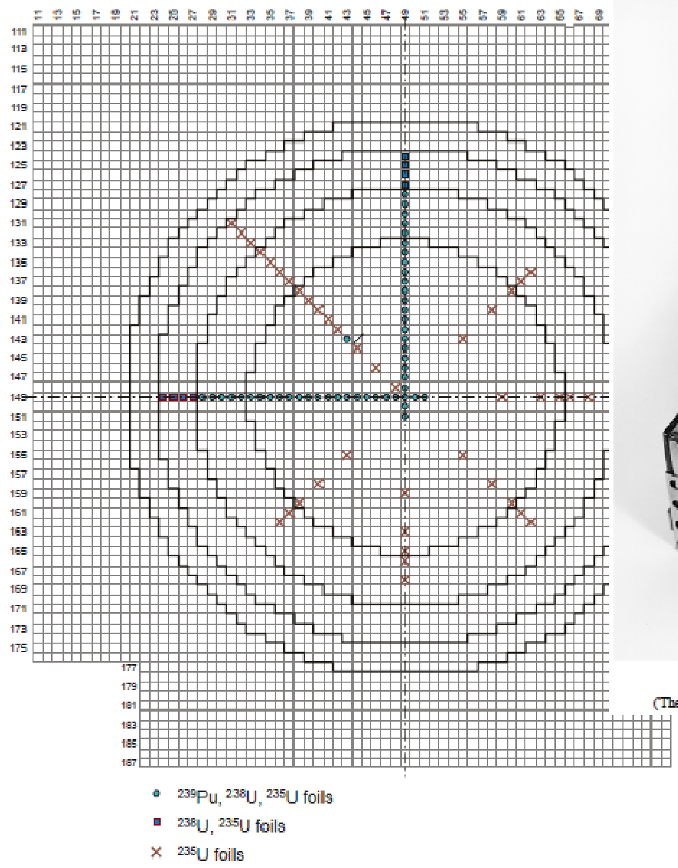


Figure 1.31. Measured Radial Reaction Rate Positions in ZPPR-9.

Figure 1.1.8. Typical Loaded Drawer.*
(The plates are elevated above the bottom of the drawer in this photograph.)

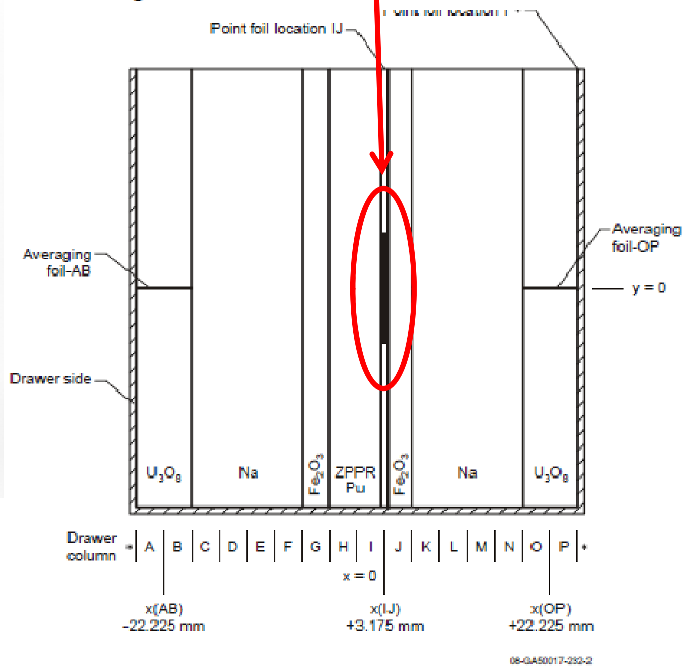
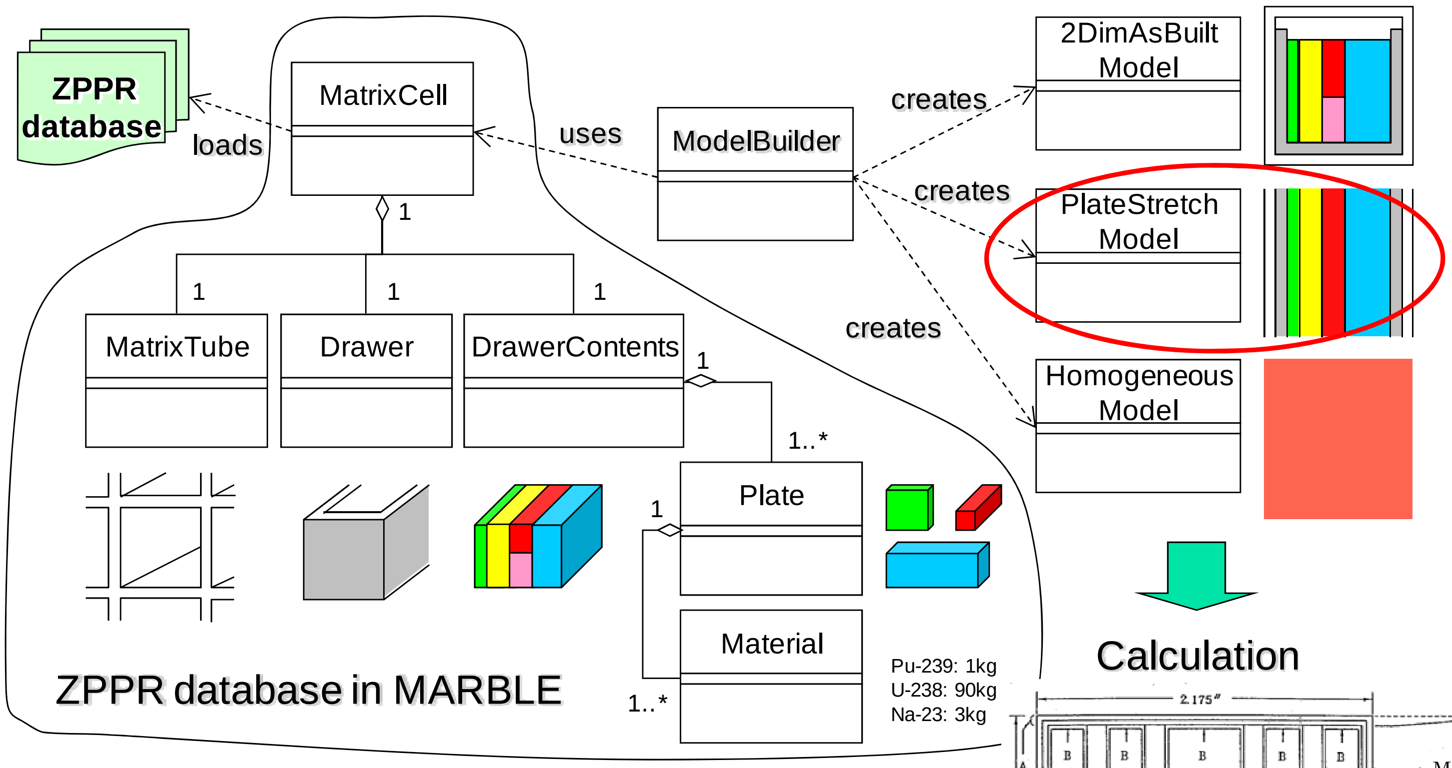
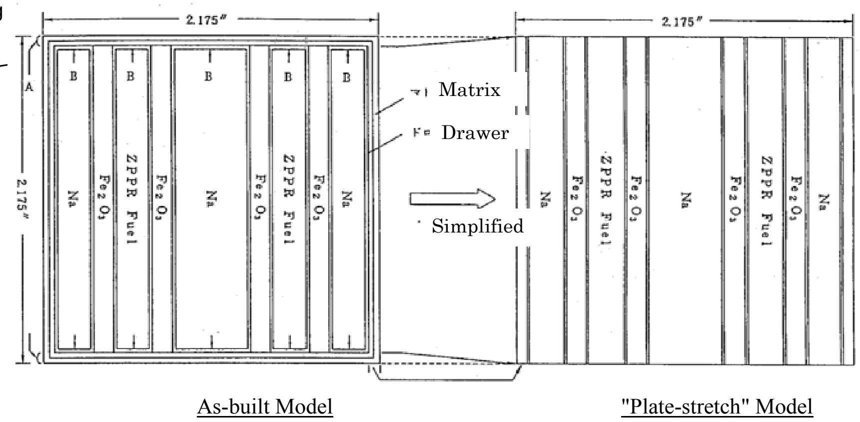


Figure 1.33. Foil Locations in a Drawer (view from front).

(Ref.) NEA IRPhE Handbook, DVD (2010)

- Keeping all as-built data and supplying the data to users
- Systematic creation of calculation models





***Note:** In **the Deterministic method**, it is easy to simulate **complicated composition distributions**, but difficult to model **3-dimensional heterogeneous structure**. In **the MC method**, it is **vice versa**.*

** First, The A and B parts in As-built model are merged into the non-fuel regions in Plate-stretch model. Next, the fuel and other plates are stretched to 2.175 inches, conserving the total atomic numbers of all isotopes.*

(Ref.) NEA IRPhE Handbook, DVD (2010)

Concluding Remarks

- **JAEA proposed how to estimate the matrix of analytical modeling error in the adjustment procedure and the following reactor design works.**

  - **If the continuous-energy Monte Carlo calculation can be applied, the error matrix will be very simple, though we have to be careful the evaluation of STD value.**

  - **When the Deterministic calculation is only available, we need some inventions and assumptions to estimate the error matrix of analytical modeling error.**

  - **A realistic approach would be to combine the Deterministic method with the MC calculation as the correction. If the simulation models to obtain the correction values are well-established from analytical modeling error viewpoint, the error matrix would be small enough with persuasiveness.**