*5th Meeting of WPEC Subgroup 33 on*
*Methods and issues for the combined use of integral experiments and covariance data*

*May 11, 2011*
*NEA Headquarters, Issy-les-Moulineaux, France*

# Simulation of Cross-section Adjustment

**Makoto ISHIKAWA**

**Japan Atomic Energy Agency (JAEA)**

# Theory of Cross-section Adjustment

* J.B.Dragt, et al.: "Methods of Adjustment and Error Evaluation of Neutron Capture Cross Sections; Application to Fission Product Nuclides," NSE 62, pp.117-129, 1977.

- Based on the Bayes theorem, i.e., the conditional probability estimation method
  → To maximize the posterior probability that a cross-section set, ***T***, is true, under the condition that the information of integral experiment, ***Re***, is obtained.

$$J(T) = (T-T_0)^t M^{-1}(T-T_0) + [Re-Rc(T)]^t[Ve+Vm]^{-1}[Re-Rc(T)]$$

Minimize the function $J(T)$. → $dJ(T)/dT = 0$

- The adjusted cross-section set ***T'***, and its uncertainty (covariance), ***M'*** ***(Algebra)***

$$T' = T_0 + MG^t[GMG^t+Ve+Vm]^{-1}[Re-Rc(T_0)]$$

$$M' = M - MG^t[GMG^t+Ve+Vm]^{-1}GM$$

✓If $GMG^t \ll Ve+Vm$, $T' \fallingdotseq T_0$ and $GM'G^t \fallingdotseq GMG^t$
✓If $GMG^t \gg Ve+Vm$, $GM'G^t \fallingdotseq Ve+Vm$
✓If $GMG^t \fallingdotseq Ve+Vm$, $GM'G^t \fallingdotseq 1/2 \times GMG^t$

- Prediction error induced by the cross-section errors

Before adjustment: $GMG^t$ After adjustment: $GM'G^t$

Where, $T_0$ : Cross-section set before adjustment
$M$ : Covariance before adjustment
$Re$ : Measured values of integral experiments
$Rc$ : Analytical values of integral experiments
$Ve$ : Experimental errors of integral experiments
$Vm$ : Analytical modeling errors of integral experiments
$G$ : Sensitivity coefficients, $(dR/R)/(d\sigma/\sigma)$

# Simulation of Cross-section Adjustment (Standard Case)

**( 3 cross-sections & 2 integral data system)**

$$Cross-section\ change:\ T' = To + MG^t\left[GMG^t + Ve + Vm\right]^{-1}\left[\mathrm{Re} - Rc(To)\right]$$

$$Covariance\ change:\ M' = M - MG^t\left[GMG^t + Ve + Vm\right]^{-1}GM$$

**Prior-covariance of cross-sections**

**M**

| | s1 | s2 | s3 |
|---|---|---|---|
| s1 | 1 | 0 | 0 |
| s2 | 0 | 1 | 0 |
| s3 | 0 | 0 | 1 |

**Sensitivity coefficients**

**G**

| | s1 | s2 | s3 |
|---|---|---|---|
| R1 | 1 | 1 | 1 |
| R2 | 1.01 | 1 | 0.99 |

**Experimental plus analytical modeling errors**

**Ve+Vm**

| | R1 | R2 |
|---|---|---|
| R1 | 0.1 | 0 |
| R2 | 0 | 0.1 |

**E-C values**

**Re-Rc**

| | |
|---|---|
| R1 | 1 |
| R2 | 1 |

**Cross-section changes by adjustment**

**T'-To**

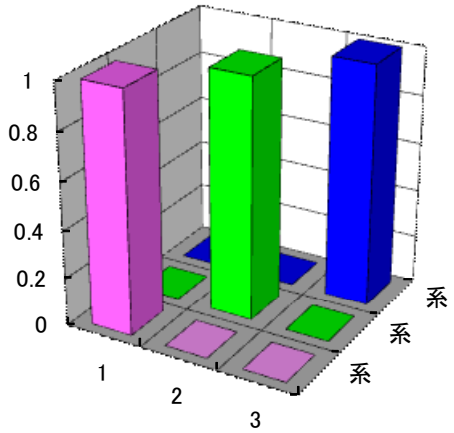
| | |
|---|---|
| s1 | 0.33 |
| s2 | 0.33 |
| s3 | 0.33 |

**Post-covariance of cross-sections**

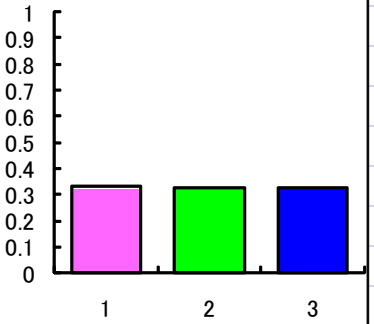
**M'**

| | s1 | s2 | s3 |
|---|---|---|---|
| s1 | 0.67 | -0.33 | -0.33 |
| s2 | -0.33 | 0.67 | -0.33 |
| s3 | -0.33 | -0.33 | 0.67 |

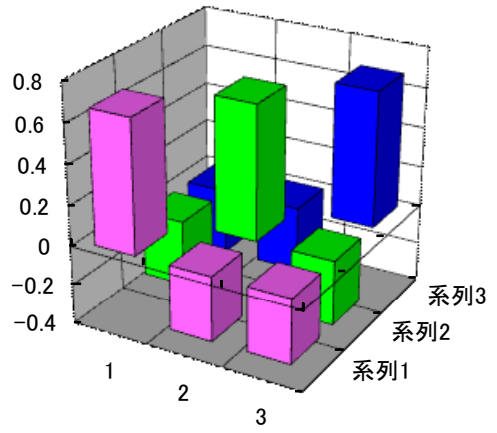
**M : Prior-covariance**




**T'-To : Cross-section change**




**M' : Post-covariance**



## Simulation of Cross-section Adjustment
## ( Case 1: Effect of Cross-section STD )

**( 3 cross-sections & 2 integral data system)**

$$Cross-section\ change:\ T' = To + MG^t\left[GMG^t + Ve + Vm\right]^{-1}\left[\mathrm{Re} - Rc(To)\right]$$

$$Covariance\ change:\ M' = M - MG^t\left[GMG^t + Ve + Vm\right]^{-1}GM$$

**Prior-covariance of cross-sections**

**M**

| | s1 | s2 | s3 |
|---|---|---|---|
| s1 | 2 | 0 | 0 |
| s2 | 0 | 1 | 0 |
| s3 | 0 | 0 | 1 |

**Sensitivity coefficients**

**G**

| | s1 | s2 | s3 |
|---|---|---|---|
| R1 | 1 | 1 | 1 |
| R2 | 1.01 | 1 | 0.99 |

**Experimental plus analytical modeling errors**

**Ve+Vm**

| | R1 | R2 |
|---|---|---|
| R1 | 0.1 | 0 |
| R2 | 0 | 0.1 |

**E-C values**

**Re-Rc**

| | |
|---|---|
| R1 | 1 |
| R2 | 1 |

**Cross-section changes by adjustment**

**T'-To**

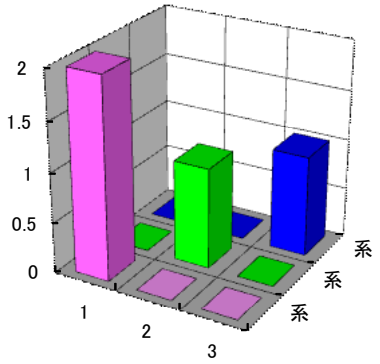
| | |
|---|---|
| s1 | 0.49 |
| s2 | 0.25 |
| s3 | 0.25 |

**Post-covariance of cross-sections**

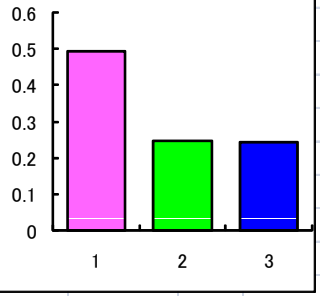
**M'**

| | s1 | s2 | s3 |
|---|---|---|---|
| s1 | 1.00 | -0.49 | -0.49 |
| s2 | -0.49 | 0.75 | -0.25 |
| s3 | -0.49 | -0.25 | 0.76 |

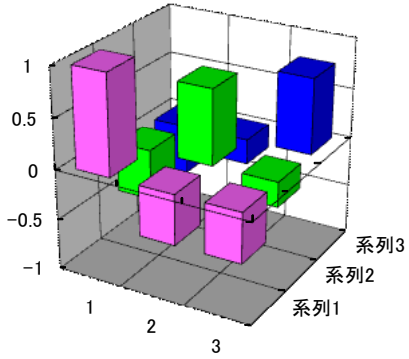
**M : Prior-covariance**




**T'-To : Cross-section change**




**M' : Post-covariance**




=> If a cross-section STD is large, the change of the cross-section by the adjustment is also large. On the other hand, too small STD is very dangerous, since it may attribute its own error to other cross-sections.

## Simulation of Cross-section Adjustment
## ( Case 2: Effect of Cross-section correlation )

**( 3 cross-sections & 2 integral data system)**

$$Cross-section\ change:\ T' = To + MG^t\left[GMG^t + Ve + Vm\right]^{-1}\left[\mathrm{Re} - Rc(To)\right]$$

$$Covariance\ change:\ M' = M - MG^t\left[GMG^t + Ve + Vm\right]^{-1}GM$$

**Prior-covariance of cross-sections**

**M**

| | s1 | s2 | s3 |
|---|---|---|---|
| s1 | 1 | 0.9 | 0 |
| s2 | 0.9 | 1 | 0 |
| s3 | 0 | 0 | 1 |

**Sensitivity coefficients**

**G**

| | s1 | s2 | s3 |
|---|---|---|---|
| R1 | 1 | 1 | 1 |
| R2 | 1.01 | 1 | 0.99 |

**Experimental plus analytical modeling errors**

**Ve+Vm**

| | R1 | R2 |
|---|---|---|
| R1 | 0.1 | 0 |
| R2 | 0 | 0.1 |

**E-C values**

**Re-Rc**

| | |
|---|---|
| R1 | 1 |
| R2 | 1 |

**Cross-section changes by adjustment**

**T'-To**

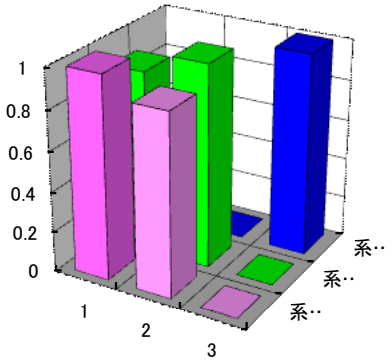
| | |
|---|---|
| s1 | 0.39 |
| s2 | 0.39 |
| s3 | 0.20 |

**Post-covariance of cross-sections**

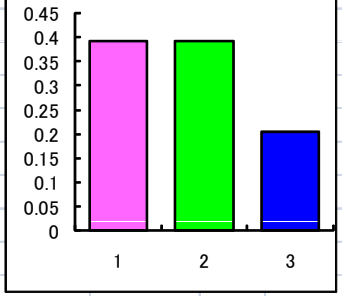
**M'**

| | s1 | s2 | s3 |
|---|---|---|---|
| s1 | 0.25 | 0.15 | -0.39 |
| s2 | 0.15 | 0.25 | -0.39 |
| s3 | -0.39 | -0.39 | 0.80 |

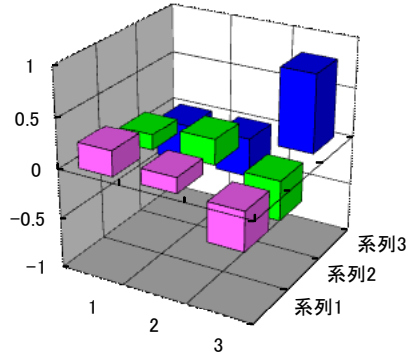
**M : Prior-covariance**




**T'-To : Cross-section change**




**M' : Post-covariance**




=> Positive correlation between cross-sections plays like large STD values of the cross-sections, if the sensitivity coefficients and other integral parameters are consistent.

## Simulation of Cross-section Adjustment
## ( Case 3: Effect of Sensitivity coefficient )

**( 3 cross-sections & 2 integral data system)**

$$Cross-section\ change:\ T' = To + MG^t\left[GMG^t + Ve + Vm\right]^{-1}\left[\mathrm{Re} - Rc(To)\right]$$

$$Covariance\ change:\ M' = M - MG^t\left[GMG^t + Ve + Vm\right]^{-1}GM$$

**Prior-covariance of cross-sections**

**M**

| | s1 | s2 | s3 |
|---|---|---|---|
| s1 | 1 | 0 | 0 |
| s2 | 0 | 1 | 0 |
| s3 | 0 | 0 | 1 |

**Sensitivity coefficients**

**G**

| | s1 | s2 | s3 |
|---|---|---|---|
| R1 | 2 | 1 | 1 |
| R2 | 2 | 1 | 0.99 |

**Experimental plus analytical modeling errors**

**Ve+Vm**

| | R1 | R2 |
|---|---|---|
| R1 | 0.1 | 0 |
| R2 | 0 | 0.1 |

**E-C values**

**Re-Rc**

| | |
|---|---|
| R1 | 1 |
| R2 | 1 |

**Cross-section changes by adjustment**

**T'-To**

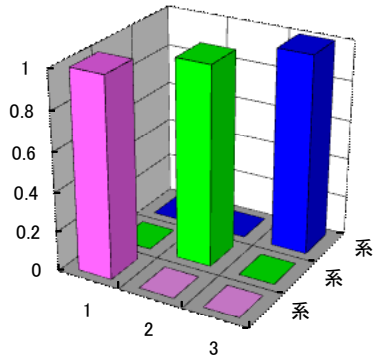
| | |
|---|---|
| s1 | 0.33 |
| s2 | 0.17 |
| s3 | 0.16 |

**Post-covariance of cross-sections**

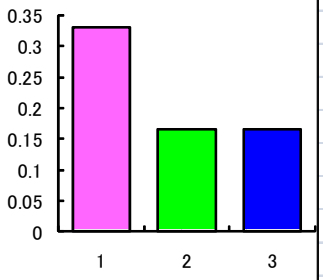
**M'**

| | s1 | s2 | s3 |
|---|---|---|---|
| s1 | 0.34 | -0.33 | -0.33 |
| s2 | -0.33 | 0.83 | -0.16 |
| s3 | -0.33 | -0.16 | 0.84 |

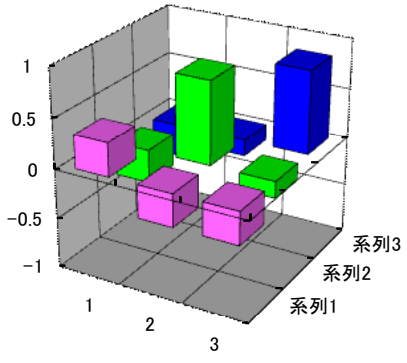
**M : Prior-covariance**




**T'-To : Cross-section change**




**M' : Post-covariance**




=> Large sensitivity coefficients work like large STD values of cross-sections, if the E-C values are consistent.

## Simulation of Cross-section Adjustment
## ( Case 4: Effect of Integral parameter errors )

**( 3 cross-sections & 2 integral data system)**

$$Cross-section\ change:\ T' = To + MG^t\left[GMG^t + Ve + Vm\right]^{-1}\left[\mathrm{Re} - Rc(To)\right]$$

$$Covariance\ change:\ M' = M - MG^t\left[GMG^t + Ve + Vm\right]^{-1}GM$$

**Prior-covariance of cross-sections**

**M**

| | s1 | s2 | s3 |
|---|---|---|---|
| s1 | 1 | 0 | 0 |
| s2 | 0 | 1 | 0 |
| s3 | 0 | 0 | 1 |

**Sensitivity coefficients**

**G**

| | s1 | s2 | s3 |
|---|---|---|---|
| R1 | 1 | 1 | 1 |
| R2 | 1.01 | 1 | 0.99 |

**Experimental plus analytical modeling errors**

**Ve+Vm**

| | R1 | R2 |
|---|---|---|
| R1 | 100 | 0 |
| R2 | 0 | 100 |

**E-C values**

**Re-Rc**

| | |
|---|---|
| R1 | 1 |
| R2 | 1 |

**Cross-section changes by adjustment**

**T'-To**

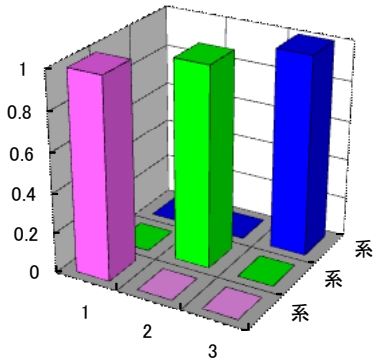
| | |
|---|---|
| s1 | 0.02 |
| s2 | 0.02 |
| s3 | 0.02 |

**Post-covariance of cross-sections**

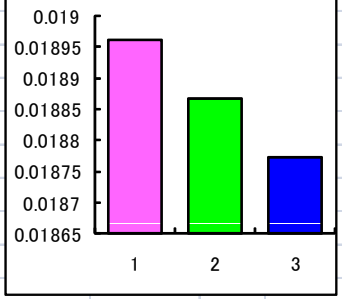
**M'**

| | s1 | s2 | s3 |
|---|---|---|---|
| s1 | 0.98 | -0.02 | -0.02 |
| s2 | -0.02 | 0.98 | -0.02 |
| s3 | -0.02 | -0.02 | 0.98 |

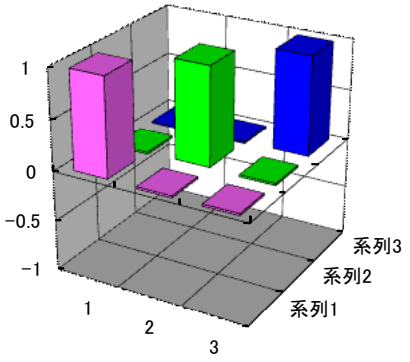
**M : Prior-covariance**




**T'-To : Cross-section change**




**M' : Post-covariance**




=> Large error values of integral parameters mean that the data have little influence, or less weight, to the adjustment results.

**Simulation of Cross-section Adjustment**
**( Case 5: Effect of C/E value )**

**( 3 cross-sections & 2 integral data system)**

$$Cross-section\ change:\ T' = To + MG^t\left[GMG^t + Ve + Vm\right]^{-1}\left[\mathrm{Re} - Rc(To)\right]$$

$$Covariance\ change:\ M' = M - MG^t\left[GMG^t + Ve + Vm\right]^{-1}GM$$

**Prior-covariance of cross-sections**

**M**

| | s1 | s2 | s3 |
|---|---|---|---|
| s1 | 1 | 0 | 0 |
| s2 | 0 | 1 | 0 |
| s3 | 0 | 0 | 1 |

**Sensitivity coefficients**

**G**

| | s1 | s2 | s3 |
|---|---|---|---|
| R1 | 1 | 1 | 1 |
| R2 | 1.01 | 1 | 0.99 |

**Experimental plus analytical modeling errors**

**Ve+Vm**

| | R1 | R2 |
|---|---|---|
| R1 | 0.1 | 0 |
| R2 | 0 | 0.1 |

**E-C values**

**Re-Rc**

| | |
|---|---|
| R1 | -1 |
| R2 | 1 |

**Cross-section changes by adjustment**

**T'-To**

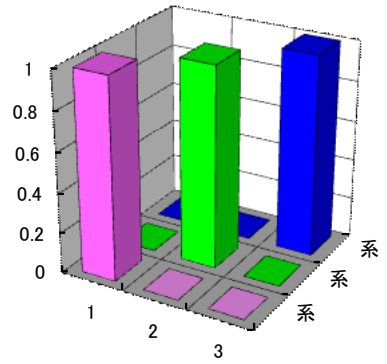
| | |
|---|---|
| s1 | 0.10 |
| s2 | 0.00 |
| s3 | -0.10 |

**Post-covariance of cross-sections**

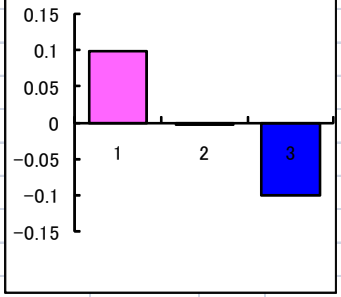
**M'**

| | s1 | s2 | s3 |
|---|---|---|---|
| s1 | 0.67 | -0.33 | -0.33 |
| s2 | -0.33 | 0.67 | -0.33 |
| s3 | -0.33 | -0.33 | 0.67 |

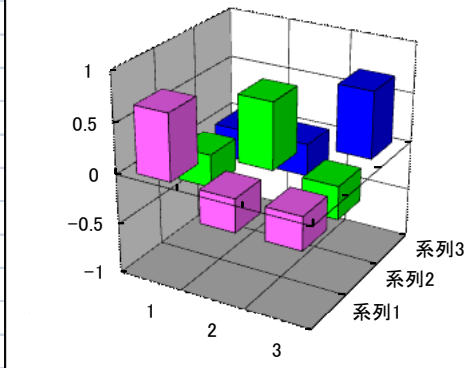
**M : Prior-covariance**




**T'-To : Cross-section change**




**M' : Post-covariance**




=> If the sensitivity coefficients and the E-C values are inconsistent with small integral error values, the results tend to be very fictitious.
However, the post-covariance is not affected at all.