

# Uncertainty Estimation of Target System Multiplication Factors with the new COMMARA Covariance Matrix

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# Introduction

Uncertainty estimation was performed for  $k_{\text{eff}}$  values of the target systems selected for the Subgroup-33 activities using the recent COMMARA covariance matrix.

The COMMARA (Covariance Multigroup Matrix for Advanced Reactor Application) covariance matrix was delivered by BNL and LANL on April 1st, 2011, following a series of revision/corrections made on the AFCI data that were initially used for uncertainty estimation of fast reactor parameters.

The target systems selected for the cross section adjustment to be performed within Subgroup 33 are:

- FBR
- ABR OXIDE STARTUP CORE
- ABR METAL
- ABR OXIDE RECYCLED CORE



## New Evaluated Covariance Matrices

Versions	Release Date	Isotope/Reaction updated with respect to the previous version
<b>AFCI-1.0</b>	November 2008	-
<b>AFCI-1.1</b>	May 2009	Pu239 (only v), Fe56, Cr50, Ni58, C, O, Na23, B10, Zr90, Mn55
<b>AFCI-1.2</b>	August 2009	U235 capture, Pu239 v and structural materials (Cr, Fe, Ni, Pb, Bi); 14 Minor Actinides were updated by Maslov review; Missing correlation matrices were recovered.
<b>AFCI-1.3</b>	April 2010	U233, U234, U235, U236, U238, Np237, Pu238, Pu239, Pu240, Pu241, Pu242, Am243, Cm242, Cm244, Fe54, Fe56, Cr52, Ni58, Na23, Mn55
<b>AFCI-2.0 first release</b>	October 2010	New evaluation
<b>AFCI-2.0 second release</b>	January 2011	Th232, Pu238, Pu240, Pu241, Am241, Am242m, Am243, Cm242, Fe54, Fe56, Cr50, Ni58, Ni60, O16, Pb208, Mg26 and Gd155
<b>COMMARA</b>	April 2011	Pu241 Fission

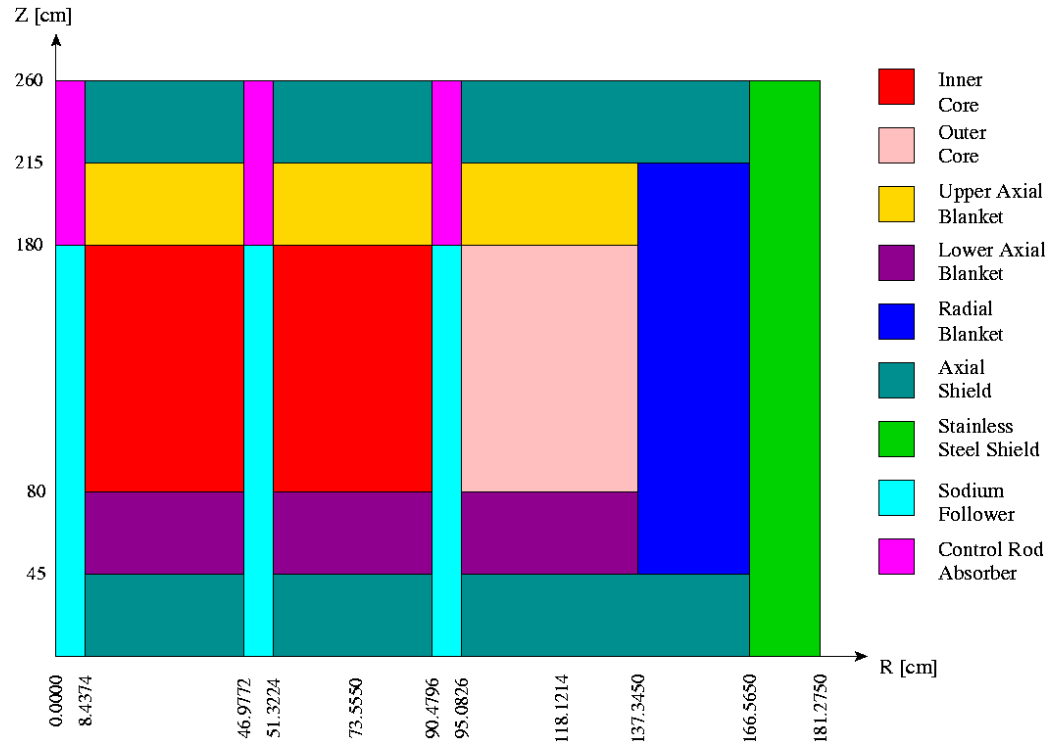


**Pu241 Fission Relative Standard Deviations (%)**

Gr.	Energy [MeV]	AFCI-1.2	AFCI-1.3	AFCI-2.0 f.r.	AFCI-2.0 s.r.	COMMARA
1	1.964E+1	10.00	0.82	10.00	0.65	1.46
2	1.000E+1	6.45	0.71	6.45	0.50	1.37
3	6.065E+0	5.00	0.67	5.00	0.45	1.39
4	3.679E+0	5.00	0.66	5.00	0.43	1.42
5	2.231E+0	5.00	0.65	5.00	0.42	1.38
6	1.353E+0	15.00	0.67	15.00	0.44	1.38
7	8.209E-1	16.24	0.68	16.24	0.46	1.27
8	4.979E-1	17.01	0.74	17.01	0.55	1.33
9	3.020E-1	18.88	0.81	18.88	0.64	1.21
10	1.832E-1	20.00	0.81	20.00	0.63	1.07
11	1.111E-1	16.89	0.84	16.89	0.68	1.11
12	6.738E-2	14.99	0.85	14.99	0.69	1.27
13	4.087E-2	11.88	0.98	11.88	0.85	1.45
14	2.479E-2	10.00	1.25	10.00	1.14	1.59
15	1.503E-2	10.00	1.78	10.00	1.71	2.15
16	9.119E-3	10.00	1.78	10.00	1.71	1.97
17	5.531E-3	10.00	2.07	10.00	2.01	2.20
18	3.355E-3	10.00	2.22	10.00	2.17	2.46
19	2.035E-3	9.97	2.07	9.97	2.01	2.47
20	1.234E-3	7.45	1.04	7.45	1.14	1.47
21	7.485E-4	5.93	5.93	5.93	5.93	1.50
22	4.540E-4	5.00	5.00	5.00	5.00	1.50
23	3.043E-4	5.00	5.00	5.00	5.00	1.50
24	1.486E-4	5.00	5.00	5.00	5.00	1.50
25	9.166E-5	5.00	5.00	5.00	5.00	1.50
26	6.790E-5	5.00	5.00	5.00	5.00	1.50
27	4.017E-5	5.00	5.00	5.00	5.00	1.50
28	2.260E-5	5.00	5.00	5.00	5.00	1.50
29	1.371E-5	5.00	5.00	5.00	5.00	1.50
30	8.315E-6	5.00	5.00	5.00	5.00	1.50
31	4.000E-6	5.00	5.00	5.00	5.00	1.50
32	5.400E-7	1.00	1.00	1.00	1.00	1.50
33	1.000E-7	1.00	1.00	1.00	1.00	1.50



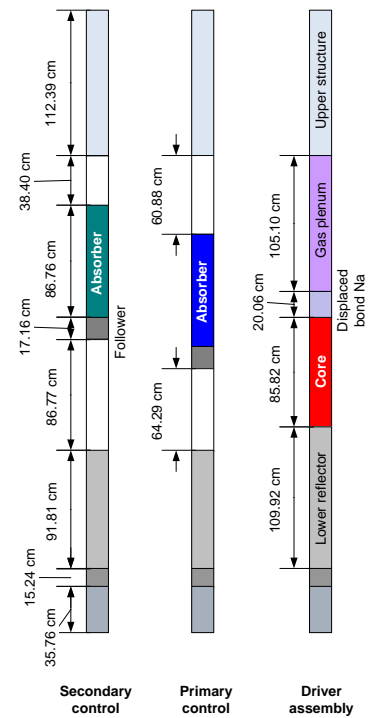
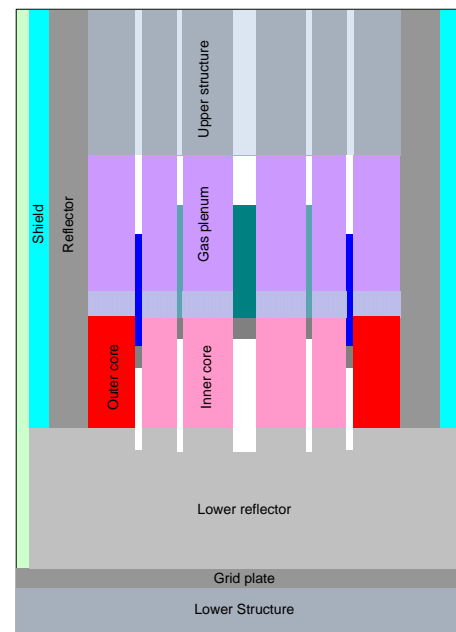
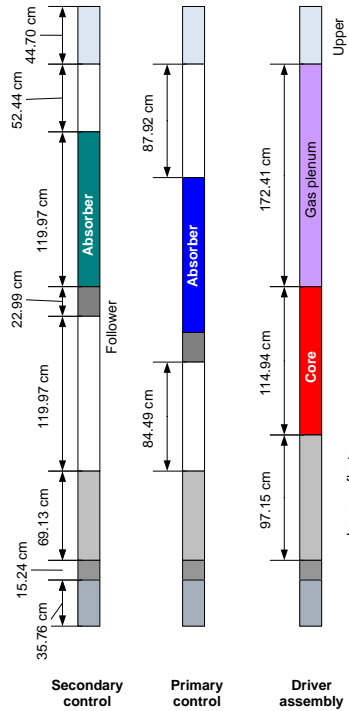
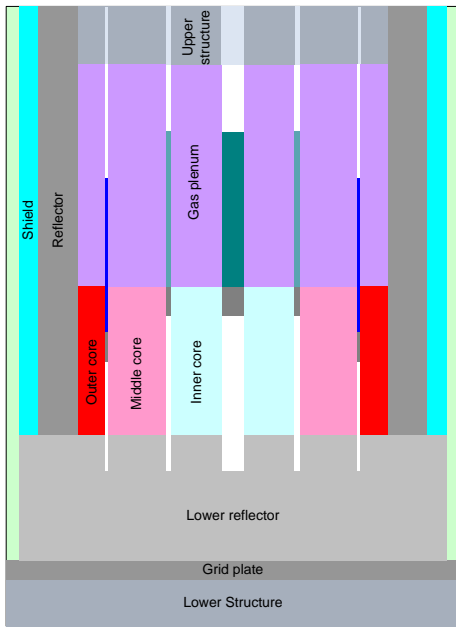
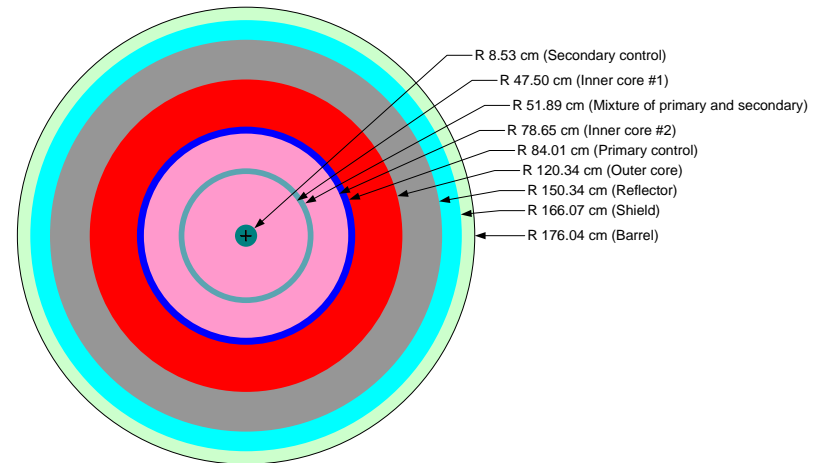
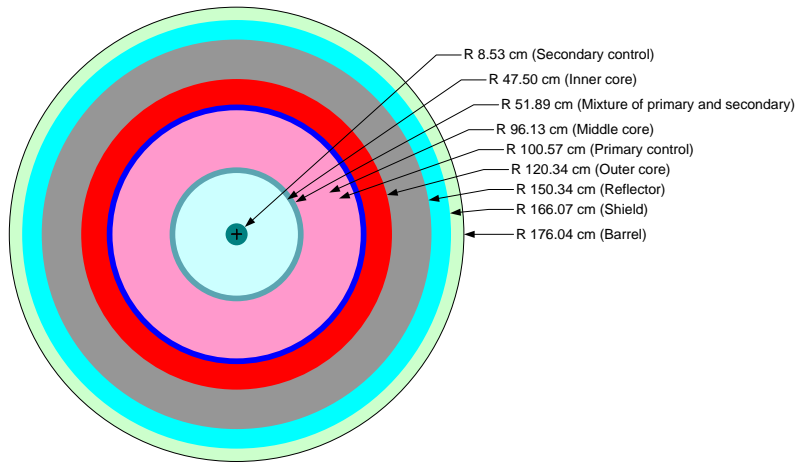
## Target Systems Selected for the Subgroup-33 Activities



**RZ Model of the FBR Reactor**



# Target Systems Selected for the Subgroup-33 Activities



RZ Model of 1000 MWt ABR Oxide Core

RZ Model of 1000 MWt ABR Metal Core



## Computational Tools

Sensitivity analysis for the target systems is still under way and calculations are performed with the following deterministic codes:

- 1) ENDF/B-VII - MC<sup>2</sup>-2 - VARI3D (diffusion theory);
- 2) ENDF/B-VII - ECCO - ERANOS2.2 (transport  $S_4P_1$ ).

The uncertainty estimation up to now has been performed for FBR, using sensitivities obtained with both VARI3D and ERANOS2.2, and for the ABRs Oxide Startup and Metal Cores using sensitivities obtained with VARI3D only. Results for ABRs Oxide Startup and Metal Cores using ERANOS and for ABR Recycled core will be available in the coming months.

NT sensitivity analysis for all experiments selected for the Subgroup-33 studies has been already performed using both VARI-3D and ERANOS codes.



# Adopted Meshes for the Spatial Calculations

## FBR

R-axis	Point	1	6	25	28	39	48	51	63	73	88	96
	Dimension [cm]	0.0000	8.4374	46.9772	51.3224	73.5550	90.4796	95.0826	118.1214	137.3450	166.5650	181.2750
	mesh width		1.6875	2.0284	1.4484	2.0211	1.8805	1.5343	1.9199	1.9224	1.9480	1.8388
Z-axis	Point	1	24	42	92	110	133					
	Dimension [cm]	0.0000	45.0000	80.0000	180.0000	215.0000	260.0000					
	mesh width		1.9565	1.9444	2.0000	1.9444	1.9565					

## ABR Oxide Core

R-axis	Point	1	6	26	29	52	55	65	80	88	93						
	Dimension [cm]	0.00	8.53	47.50	51.89	96.13	100.57	120.34	150.34	166.07	176.04						
	mesh width		1.71	1.95	1.46	1.92	1.48	1.98	2.00	1.97	1.99						
Z-axis	Point	1	19	27	62	76	88	100	105	112	117	124	136	178	196	222	245
	Dimension [cm]	0.00	35.76	51.00	120.13	148.15	171.14	194.13	204.62	217.11	227.61	240.10	263.09	347.58	383.06	435.50	480.20
	mesh width		1.99	1.91	1.98	2.00	1.92	1.92	2.10	1.78	2.10	1.78	1.92	2.01	1.97	2.02	1.94

## ABR Metal Core

R-axis	Point	1	6	26	29	43	46	64	79	87	92							
	Dimension [cm]	0.00	8.53	47.50	51.89	78.65	84.01	120.34	150.34	166.07	176.04							
	mesh width		1.71	1.95	1.46	1.91	1.79	2.02	2.00	1.97	1.99							
Z-axis	Point	1	19	27	73	82	91	100	106	109	115	118	127	137	160	171	190	247
	Dimension [cm]	0.00	35.76	51.00	142.81	160.92	178.08	195.25	207.10	212.41	224.26	229.58	246.74	266.80	311.02	333.50	371.90	484.29
	mesh width		1.99	1.91	2.00	2.01	1.91	1.91	1.98	1.77	1.98	1.77	1.91	2.01	1.92	2.04	2.02	1.97



### Multiplication Factor Calculated Values for the Target Systems of Subgroup-33

Parameters	FBR	ABR Oxide Core Startup	ABR Oxide Core Recycled	ABR Metal Core
ANL DIF3D Diffusion	1.05425	1.04164	1.00597	- <sup>(a)</sup>
ANL ERANOS S <sub>4</sub> P <sub>1</sub>	1.05408	-	-	-

<sup>(a)</sup> Not Available yet



## Different way to present uncertainty contributions

The COMMARA data include only partial correlation data of two different reactions of the same isotope. Thus, the variation of the parameter R due to the cross section uncertainties of the isotope i can be written as:

$$\left( \frac{(\Delta R)^2}{R^2} \right)_i = \sum_k S_k^T V_k S_k + 2 \sum_{k>l} \sum_l S_k^T V_{kl} S_l$$

where  $S_k$  is the sensitivity vector of cross section  $k$ ,  $V_k$  is the covariance matrix associated with the cross section  $k$ ,  $V_{kl}$  is the covariance matrix of cross sections  $k$  and  $l$ .

Uncertainty contributions of the single isotope cross sections will be presented as:

$$\left( \frac{\Delta R}{R} \right)_{i,k} = \sqrt{S_k^T V_k S_k}$$

For the contribution of two correlated cross sections, the following correlation is given:

$$\left( \frac{(\Delta R)^2}{R^2} \right)_{i,kl} = \frac{S_k^T V_{kl} S_l}{\sqrt{S_k^T V_k S_k} \cdot \sqrt{S_l^T V_l S_l}}$$



## Different way to present uncertainty contributions

### Numerical example: case of FBR multiplication factor with COMMARA data

Using VARI3D sensitivities:

$$u1 = \left( \frac{(\Delta Void)^2}{V^2} \right)_{U8\_el} = \sqrt{S_{U8\_el}^T \cdot V_{U8\_el} \cdot S_{U8\_el}} = 0.08\%$$

$$u2 = \left( \frac{(\Delta Void)^2}{V^2} \right)_{U8\_inel} = \sqrt{S_{U8\_inel}^T \cdot V_{U8\_inel} \cdot S_{U8\_inel}} = 0.74\%$$

$$u12 = \left( \frac{(\Delta Void)^2}{V^2} \right)_{U8\_el\_inel} = \frac{S_{U8\_el}^T \cdot V_{U8\_el\_inel} \cdot S_{U8\_inel}}{\sqrt{S_{U8\_el}^T \cdot V_{U8\_el} \cdot S_{U8\_el} \cdot S_{U8\_inel}^T \cdot V_{U8\_inel} \cdot S_{U8\_inel}}} = 91.97\%$$



The uncertainty on void worth due to U8 elastic and U8 inelastic combined together would be:

$$\sqrt{u1^2 + u2^2 + 2 \cdot u12 \cdot u1 \cdot u2} = \sqrt{0.0008^2 + 0.0074^2 + 2 \cdot 0.9197 \cdot 0.0008 \cdot 0.0074} = 0.81\%$$



**Major Single and Two Correlated Cross Section Contributions (%) to the  $k_{eff}$  Total Uncertainty of FBR Estimated with the COMMARA Matrix**

<b>FBR</b>	<b>VARI3D</b>	<b>ERANOS</b>
<b>Major Single Cross Section Uncertainties (%)</b>		
U238 Fission	0.04	0.04
U238 Nu	0.13	0.14
U238 Capture	0.30	0.32
U238 Elastic	0.08	0.04
U238 Inelastic	0.74	0.77
Pu239 Fission	0.17	0.18
Pu239 Nu	0.05	0.06
Pu239 Capture	0.23	0.24
Pu239 Inelastic	0.07	0.07
Pu240 Fission	0.10	0.10
Pu240 Nu	0.18	0.19
Pu240 Capture	0.12	0.13
Pu240 Inelastic	0.04	0.04
Pu241 Fission	0.08	0.08
Pu241 Nu	0.04	0.04
Pu241 Capture	0.10	0.10
Pu242 Fission	0.03	0.04
Pu242 Capture	0.09	0.09
Fe56 Capture	0.08	0.09
Fe56 Elastic	0.06	0.04
Fe56 Inelastic	0.10	0.10
Ni58 Capture	0.03	0.08
Na Inelastic	0.07	0.08
O Capture	0.00	0.13
O Elastic	0.05	0.05
<b>Major Two Correlated Cross Section Uncertainty Contribution Coefficients (%)</b>		
U238 Inelastic Elastic	91.97	75.50
<b>Total Uncertainty</b>	<b>0.99</b>	<b>1.01</b>



**Major Single and Two Correlated Cross Section Contributions (%) to the  $k_{\text{eff}}$  Total Uncertainty of ABR Oxide Startup Estimated with the COMMARA Matrix**

<b>ABR OXIDE STARTUP</b>	<b>VARI3D</b>
<b>Major Single Cross SectionmUncertainties (%)</b>	
U238 Fission	0.03
U238 Nu	0.12
U238 Capture	0.28
U238 Elastic	0.06
U238 Inelastic	0.55
Pu239 Fission	0.23
Pu239 Nu	0.07
Pu239 Capture	0.31
Pu239 Inelastic	0.07
Pu240 Fission	0.05
Pu240 Nu	0.08
Pu240 Capture	0.06
Fe54 Capture	0.02
Fe54 Elastic	0.03
Fe56 Capture	0.16
Fe56 Elastic	0.29
Fe56 Inelastic	0.15
Cr52 Elastic	0.06
Na Elastic	0.04
Na Inelastic	0.07
O Elastic	0.03
<b>Major Two Correlated Cross Section Uncertainty Contribution Coefficients (%)</b>	
U238 Inelastic Elastic	88.15
Pu239 Capture Fission	-1.05
Pu239 Inelastic Elastic	89.55
Fe56 Inelastic Elastic	7.17
<b>Total Uncertainty</b>	<b>0.88</b>



**Major Single and Two Correlated Cross Section Contributions (%) to the  $k_{\text{eff}}$  Total  
Uncertainty of ABR Metal Estimated with the COMMARA Matrix**

<b>ABR METAL</b>	<b>VARI3D</b>
<b>Major Single Cross Section Uncertainties (%)</b>	
U238 Fission	0.04
U238 Nu	0.15
U238 Capture	0.22
U238 Elastic	0.09
U238 Inelastic	0.74
Pu239 Fission	0.25
Pu239 Nu	0.08
Pu239 Capture	0.23
Pu239 Elastic	0.02
Pu239 Inelastic	0.09
Pu240 Fission	0.04
Pu240 Nu	0.07
Pu240 Capture	0.03
Fe54 Capture	0.02
Fe54 Elastic	0.04
Fe56 Capture	0.15
Fe56 Elastic	0.47
Fe56 Inelastic	0.16
Cr52 Capture	0.02
Cr52 Elastic	0.11
Cr52 Inelastic	0.02
Na Elastic	0.06
Na Inelastic	0.07
Zr90 Capture	0.02
Zr90 Elastic	0.02
Zr92 Capture	0.02
<b>Major Two Correlated Cross Section Uncertainty Contribution Coefficients (%)</b>	
U238 Inelastic Elastic	86.30
Pu239 Inelastic Elastic	91.33
Fe56 Inelastic Elastic	6.93
<b>Total Uncertainty</b>	<b>1.09</b>

