A few remarks on the « a-posteriori » covariance matrix

M.Salvatores

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The global « a-priori » correlation matrix B_y has the form:

$$B_y = \begin{bmatrix} B_\sigma & 0 \\ 0 & B_E \end{bmatrix}$$

where B_σ is the «a-priori » correlation matrix of the parameters and B_E is the «a-priori » correlation matrix of the experiments. That form corresponds to the hypothesis that «a-priori» there are no correlations between parameters and experiments.

The « a-posteriori » correlation matrix (i.e. after adjustment):

$$B_{\tilde{y}} = (I - B_y A^T G^{-1} A) B_y$$

A is the sensitivity matrix with dimension $(N_p+N_E)xN_E=rxq$ where N_p is the total number of parameters (e.g. x-sections) and N_E is the total number of experiments:

$$A = \begin{vmatrix} A_{11} & A_{12} & \ldots\ldots & A_{1r} \\ A_{21} & A_{22} & \ldots\ldots & A_{2r} \\ \vdots & \vdots & & \vdots \\ A_{q1} & A_{q2} & \ldots\ldots & A_{qr} \end{vmatrix}$$

The matrix A can be rewritten as a vector with two components, each being a matrix:

$$A = \left| A_\sigma , A_E \right| = \left| S, 1 \right|$$

where A_σ (dimension $N_p x N_E$) is the sensitivity matrix of the integral experiments with respect to the parameters and A_E is a square identity matrix (dimension $N_E x N_E$).

Finally:

$$G = A B_y A^T$$

with dimension $N_E x N_E$

$$G^{-1} = \left(S^T B_\sigma S + B_E \right)^{-1}$$

The form of G^{-1} means that the « a-posteriori » correlation matrices for both parameters and experiments will contain terms depending on both Bσ and BE

$$B_{\tilde{y}} = \begin{vmatrix} B_{\tilde{\sigma}} & B_{\tilde{\sigma}\tilde{E}} \\ B_{\tilde{E}\tilde{\sigma}} & B_{\tilde{E}} \end{vmatrix}$$

$$B_{\tilde{\sigma}} = B_{\sigma} - A^{T}_{\sigma} B_{\sigma} G^{-1} A_{\sigma} B_{\sigma}$$

$$B_{\tilde{E}} = B_{E} - B_{E} G^{-1} B_{E}$$

A-posteriori Correlation matrices

$$B_{\tilde{E}\tilde{\sigma}} = B_{\tilde{\sigma}\tilde{E}} = B_{E} S G^{-1} B_{\sigma}$$

Terms of experiment/parameter correlation