

## 4. Sensitivity studies and issues

(compiled by Sandro Pelloni, PSI)

### 4.1. Participants and methodologies

#### 4.1.1 Participants

The following institutes, namely JAEA, INL, ANL, CEA and PSI, have provided full sets of unadjusted analytical data in the specified format. Sensitivity coefficients for the effective multiplication factor  $k$  have also been generated in a consistent manner by IJS, IRSN and KAERI.

In addition,

Based upon SCALE6.1 [1], ORNL has produced for JEZEBEL and FLATTOP, 238 group values by using the stochastic code TSUNAMI-1D in conjunction with ENDF/B-VII.0 data

Starting from the so called Total Monte Carlo approach, NRG has generated sensitivity coefficients for the effective multiplication factor of the seven benchmark systems based upon an innovative stochastic methodology currently under development. The first steps of this approach were to produce thousands of TALYS-based evaluated files using a stochastic sampling of nuclear parameters and benchmark all these files with simulations of integral experiments [2].

CIAE has used in conjunction with CENDL-3.1 and JENDL-4.0 data an in-house version of the SEN1D code which is currently under development, to determine the sensitivity coefficients for  $k$  of JEZEBEL and FLATTOP [3].

The ORNL data is not part of this compilation since it cannot easily be compared with the other results, the specified 33 groups being not a subset of the 238 group structure. Due to their preliminary nature, the NRG and CIAE studies were also not included.

#### 4.1.2 Methodologies

(1) For  $k$ , deterministic values of the sensitivity coefficients have been determined based on Standard Perturbation-Theory (SPT) techniques (see e.g. [4], pages 17 and 31) using transport-theory. More precisely, the various analyses were carried out on the basis of

The SAGEP code [5], in conjunction with JENDL-4 data at JAEA;

ERANOS [6] in conjunction with ENDF/B-VII.0 data at INL and ANL and

ERANOS, respectively in conjunction with JEFF-3.1.1 and JEFF-3.1.0 data, at CEA and PSI;

In addition, on the basis of ENDF/B-VII.0 data, DANTSYS [7] has been used in conjunction with the in-house code APSTRACT ('09) at KAERI, and also with SUS3D [8] at IJS.

Stochastic values of the sensitivity coefficients have been obtained at IRSN by using TSUNAMI-1D/TSUNAMI-3D [9] in conjunction with ENDF/B-VII.0 data. The sensitivity coefficients, in this case, account that a change of a given cross-section may also influence the other cross-sections through modifications of their shielding factors. More precisely, the TSUNAMI sensitivity coefficients write with the total instead of the partial derivatives, thus as

$$S_{\Sigma_{x,g}^i} = \frac{dk}{k} \frac{d\Sigma_{x,g}^i}{\Sigma_{x,g}^i} = \frac{\partial k}{k} \frac{\partial \Sigma_{x,g}^i}{\Sigma_{x,g}^i} + \sum_j \sum_y \sum_h \left( \left( \frac{\partial k}{k} \frac{\partial \Sigma_{y,h}^j}{\Sigma_{y,h}^j} \right) \left( \frac{\partial \Sigma_{y,h}^j}{\Sigma_{y,h}^j} \frac{\partial \Sigma_{x,g}^i}{\Sigma_{x,g}^i} \right) \right)$$

In the formula for these total sensitivity coefficients, the space variable has been omitted and e.g.  $\sum_{x,g}^i$  is the cross-section of nuclide  $i$  for reaction  $x$  in energy group  $g$ . The first expression on the right hand side corresponds to the standard definition, with the additional summations as an indirect term.

(2) For the reaction rates at core center relative to  $^{235}\text{U}$  fission (F25), i.e. F49/F25, F28/F25, F37/F25 and C28/F25, Generalized Perturbation Theory (GPT) (see e.g. [4], pages 18 and 32) has been consistently used in the deterministic calculations for calculating their sensitivity coefficients, and correspondingly

(3) Equivalent Generalized Perturbation Theory (EGPT) in ERANOS terminology (see e.g. [4], pages 28 and 35) has been employed for determining the sensitivity coefficients of the void reactivity effects in ZPPR9 (Na void (Step 3) and Na void (Step 5)).

## 4.2 Results

The large amount of provided analytical data has been scored for any sensitivity coefficient of a given nuclide, reaction and energy group, in terms of (a) its relative difference from the average value among the participants and (b) its relative contribution to the sum over the 33 groups of the coefficients for the same nuclide and reaction, and this in terms of individual absolute values. In order to characterize discrepant enough data in view of identifying trends, a choice was made according to the result of this score, namely only those sensitivity coefficients exceeding the arbitrary values of 50% for (a) and 20% for (b) and consequently their associated energy profiles were considered. The corresponding main findings are summarized below, the sensitivity coefficients of the fission spectrum, delayed data and average cosine of the scattering angle being so far not addressed due to the rather scarce availability of analytical data.

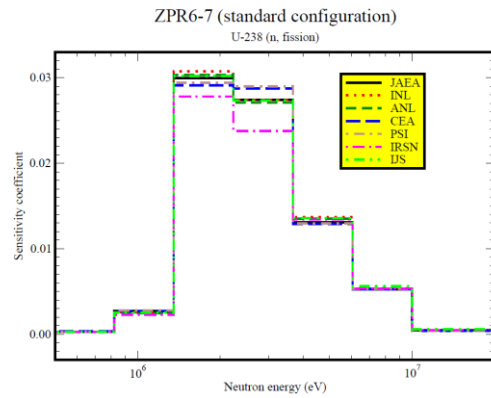
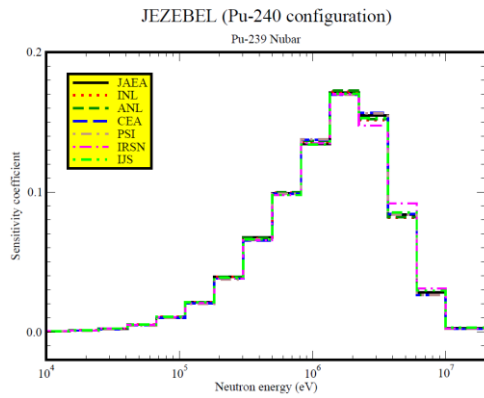
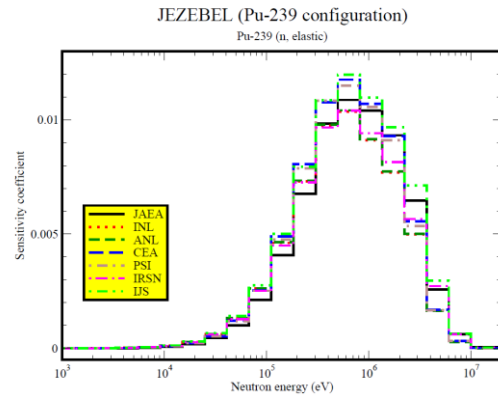
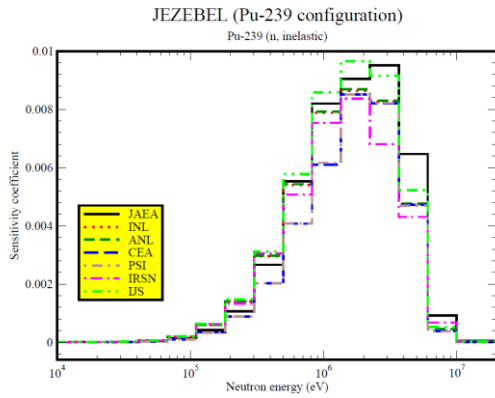
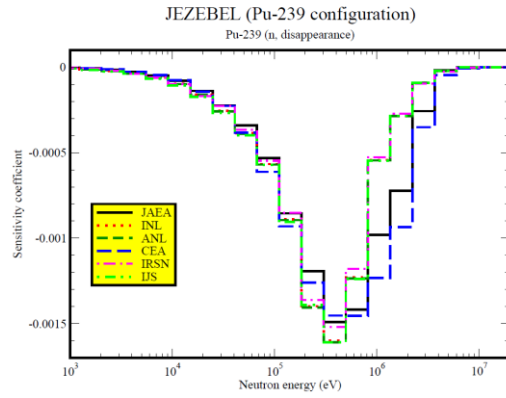
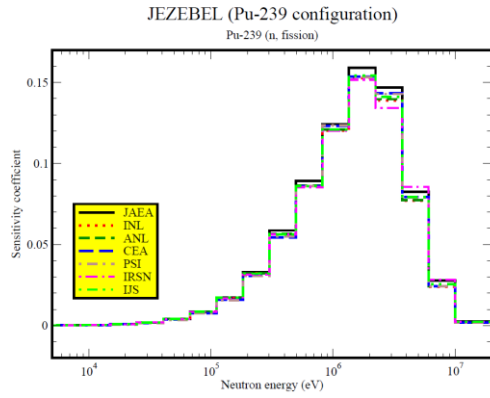
### 4.2.1 Effective multiplication factor

JAEA, INL, ANL, CEA, PSI, IJS and KAERI have provided for the benchmark systems JEZEBEL, ( $^{239}\text{Pu}$  and  $^{240}\text{Pu}$  configurations), FLATTOP ( $^{239}\text{Pu}$  configuration), ZPR6-7 (standard configuration and high  $^{240}\text{Pu}$  content), ZPPR9 and JOYO deterministic solutions based upon the SPT methodology, whereas IRSN has used for all these configurations a stochastic methodology to derive correspondingly total sensitivity coefficients (see Section 4.1.2).

By separating out the KAERI solution, which will be shortly discussed at the end of this section due to much larger individual differences, the analytical data is conveniently grouped as follows:

First considered are the sensitivity coefficients of the actinides referring to the two JEZEBEL bare spheres and to the sodium-cooled systems i.e. the two ZPR6-7 cores, ZPPR9, and JOYO. For the sodium-cooled systems, the scattering reactions are systematically excluded in this group of analytical data.

Overall consistence is shown among the participants, as illustratively shown in Fig. 1.



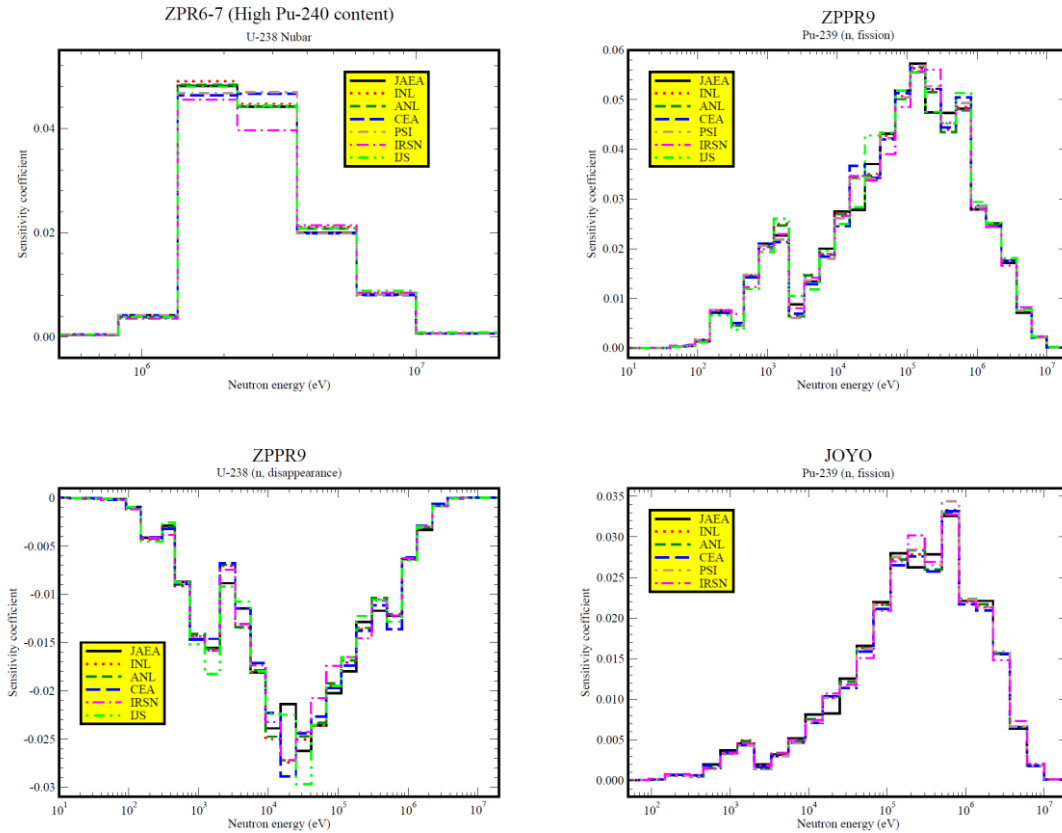
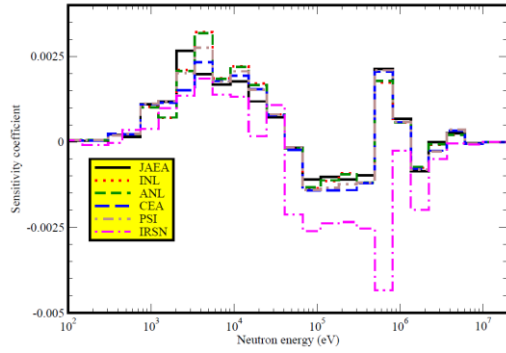


Fig. 1: Matching sensitivity profiles of the effective multiplication factor for actinides

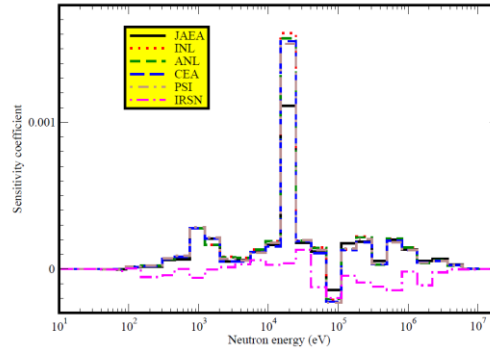
The sensitivity coefficients, in these cases, are almost independent of the code (deterministic SAGEP, ERANOS and DANTSYS/SUSD3D, stochastic TSUNAMI-1D/TSUNAMI-3D) and basic nuclear data (JENDL-4, ENDF/B-VII.0 or JEFF-3.1) being used. In particular, as a result of the shown agreement of TSUNAMI with the other codes, the indirect term appears small, confirming that resonance shielding effects, as expected, are largely unimportant in these cases.

The second group of analytical data refers to the sodium-cooled systems including those sensitivity coefficients so far left out for these systems in the first group, namely the sensitivity coefficients for the structural materials, sodium, oxygen, and those for the scattering reactions of the actinides (see Fig. 2).

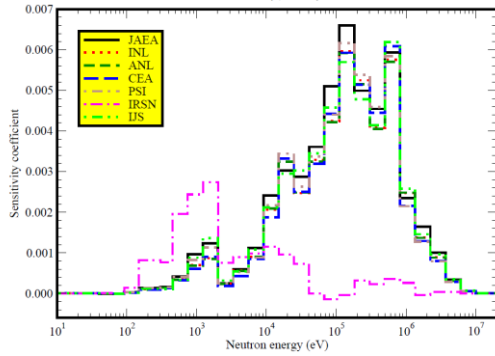
ZPR6-7 (standard configuration)  
Na-23 (n,elastic)



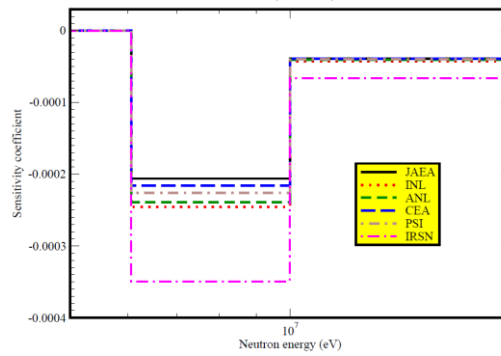
ZPR6-7 (standard configuration)  
Ni-58 (n, elastic)



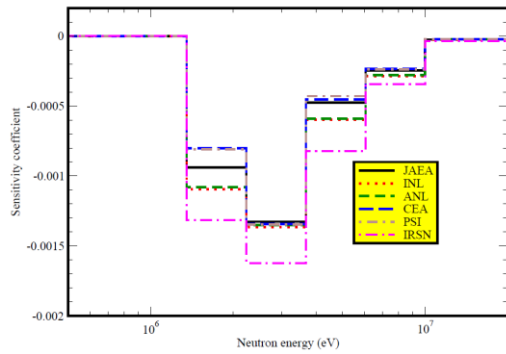
ZPR6-7 (standard configuration)  
U-238 (n, elastic)



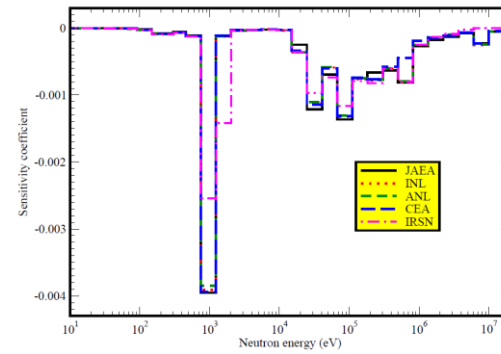
ZPR6-7 (standard configuration)  
O-16 (n, inelastic)



ZPR6-7 (High Pu-240 content)  
Cr-52 (n, inelastic)



ZPPR9  
Fe56 (n, disappearance)



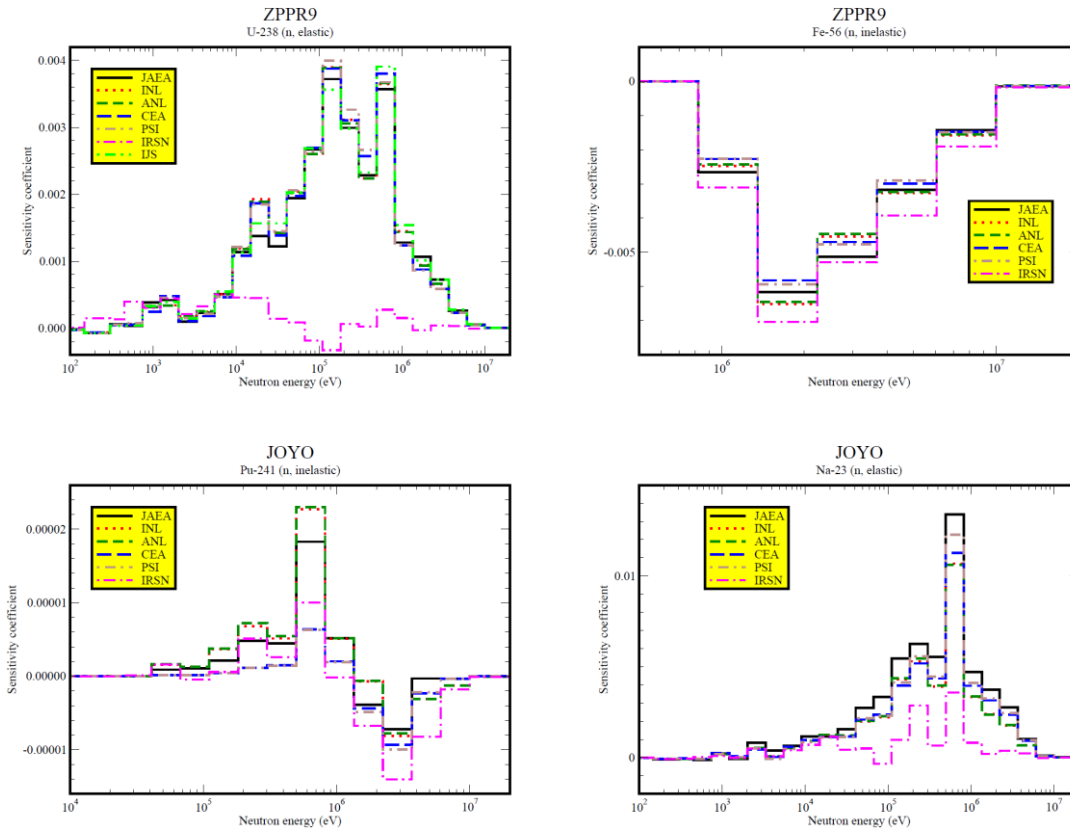


Fig. 2: Sodium-cooled systems: Representative sensitivity profiles of the effective multiplication factor for structural materials, oxygen, sodium and scattering reactions of actinides

Except for the IRSN solution (in magenta), which in several cases largely differs from the others, consistence is shown. Correspondingly, the indirect term of the total sensitivity coefficients might be quite large, in these cases. This preliminary conclusion, however, needs to be substantiated.

The sensitivity coefficients referring to the FLATTOP core form the third and last group of data. Some of the energy profiles are namely characterized by a more heterogeneous spread as compared to those belonging to the other two groups, as representatively shown in Fig. 3-1 and in Fig. 3-2 respectively for the core and reflector region. Not knowing the details of the individual calculations, understanding the causes for these differences is uneasy, and thus it is judged out of scope for this study.

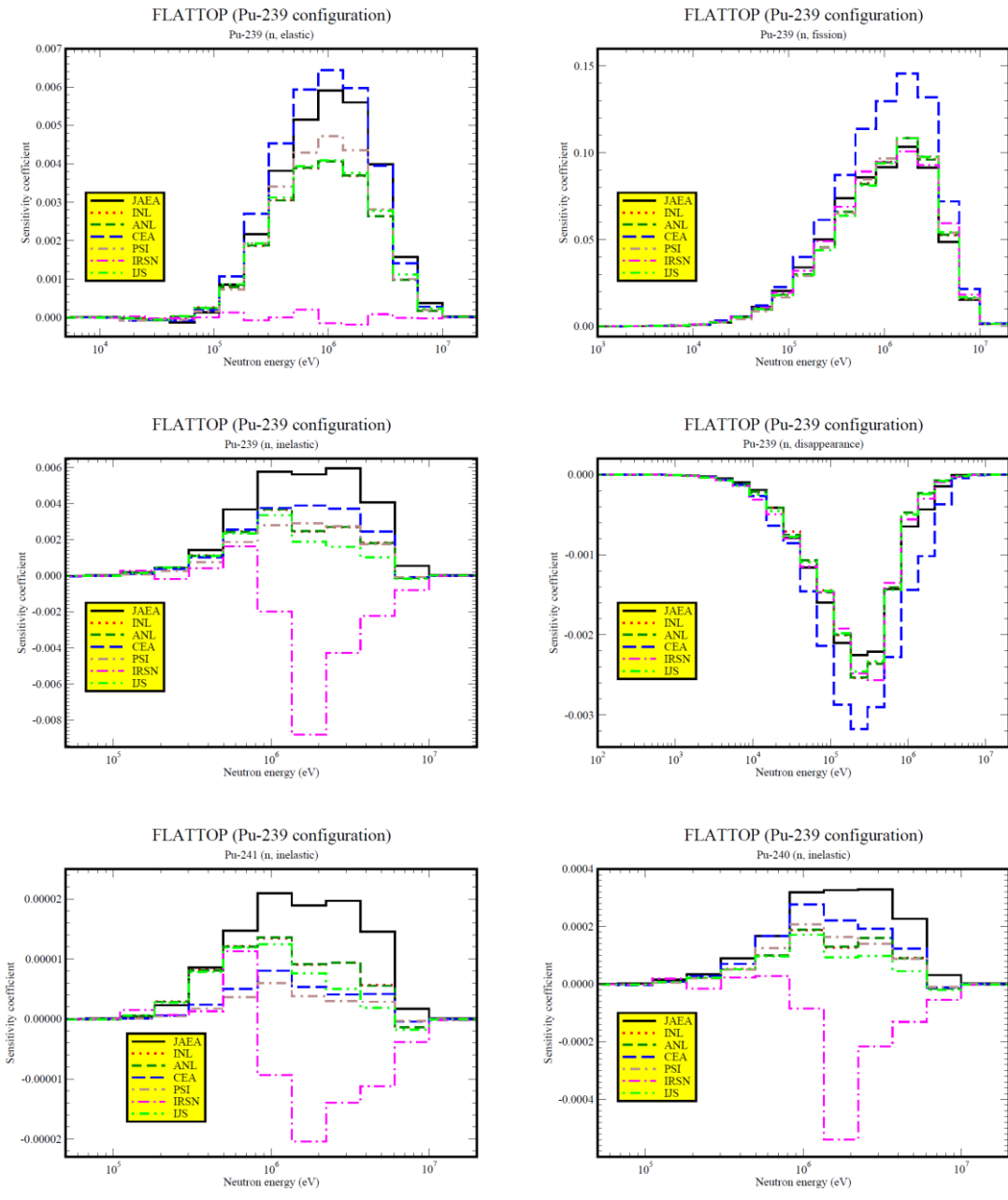


Fig. 3.1: FLATTOP (<sup>239</sup>Pu configuration): Deviating sensitivity profiles of the effective multiplication factor for Pu isotopes

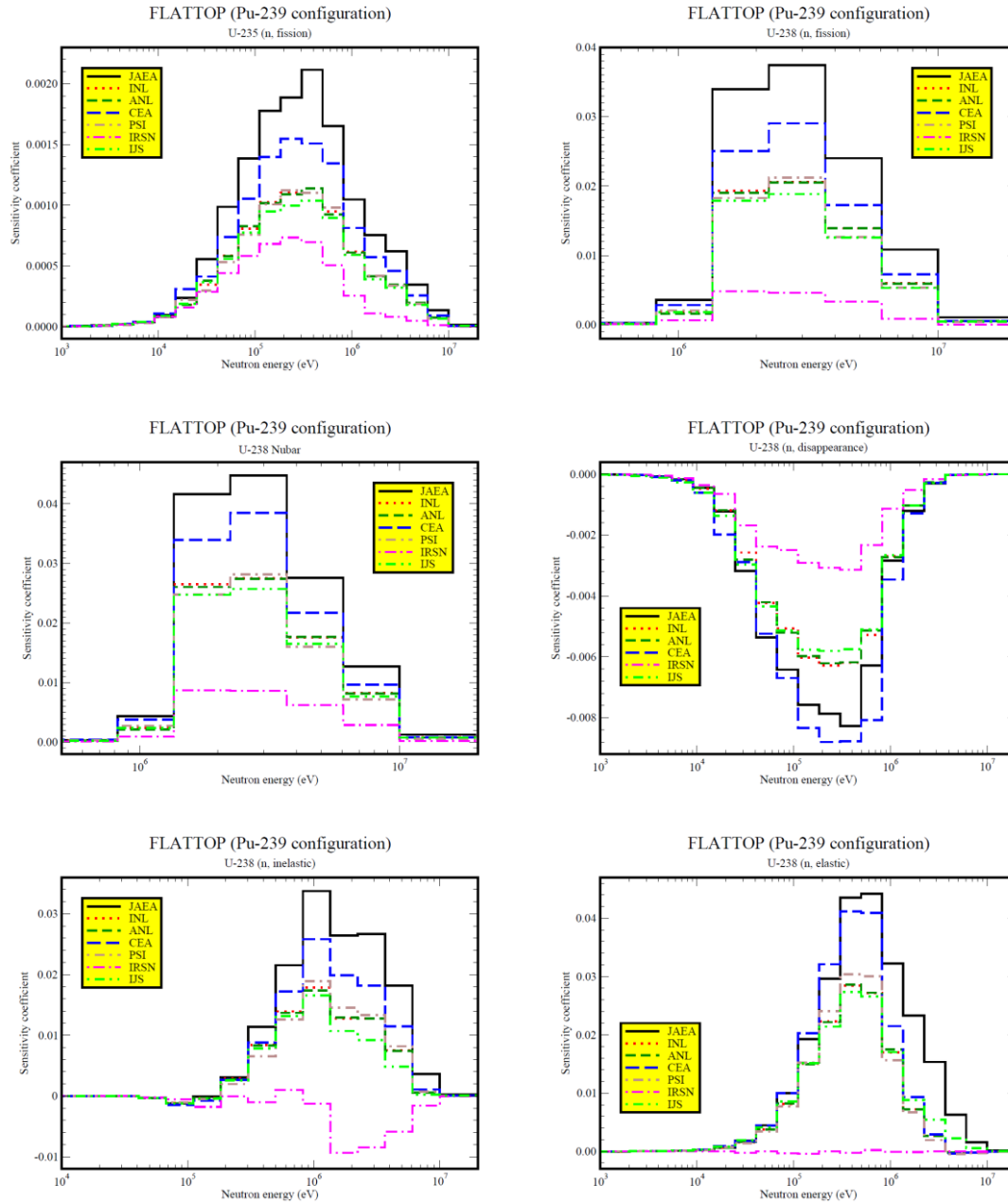


Fig. 3.2: FLATTOP ( $^{239}\text{Pu}$  configuration): Deviating sensitivity profiles of the effective multiplication factor for uranium

In several cases the JAEA and IRSN profiles (respectively in black and magenta in the two figures above) are the limiting curves. In the MeV range, the JAEA inelastic scattering coefficients are positive, whereas the corresponding IRSN values might be strongly negative. For the elastic scattering reaction of actinides, as opposed to the other solutions, the IRSN values are nearly zero in the whole energy range. The ERANOS solutions using the same ENDF/B-VII.0 data (INL and ANL), as expected, are in excellent agreement; they also agree with the corresponding IJS solution, but more or less strongly deviate from the corresponding ERANOS solutions based upon JEFF-3 data (CEA and PSI) which in turn are not as consistent among them as one would expect (compare the curves in blue with those in brown).

As anticipated, the KAERI solution often exhibits deviating or stronger values, as indicated in Fig. 4 (curves in violet), clearly requiring additional explanatory work.

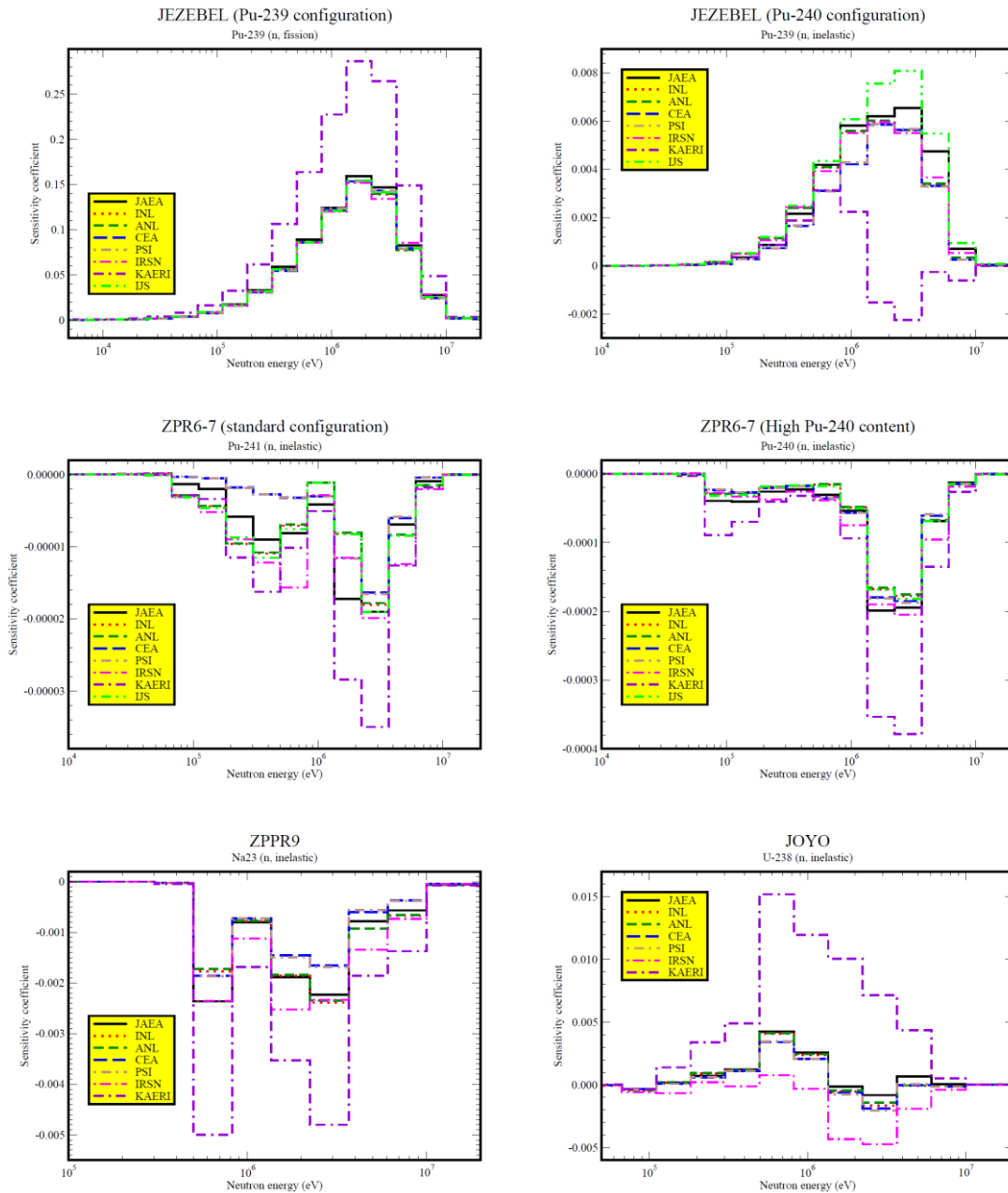


Fig. 4: Illustrative sensitivity profiles including the KAERI solution

#### 4.2.2 F49/F25

Among the participants (JAEA, INL, ANL, CEA, and PSI) providing deterministic solutions for JEZEBEL, ZPR6-7 (standard configuration) and ZPPR9 based upon the GPT methodology, consistence of the sensitivity coefficients including those for the structural materials, oxygen and sodium available in the case of ZPR6-7 and ZPPR9 only (see Fig. 5.1), is shown, with the exceptions of  $\nu$  in general and inelastic

scattering of  $^{241}\text{Pu}$ . In particular, it is seen (Fig. 5.2) that for JEZEBEL the sensitivity coefficient for  $\nu$  of  $^{239}\text{Pu}$  is nearly zero in the case of the JAEA solution.

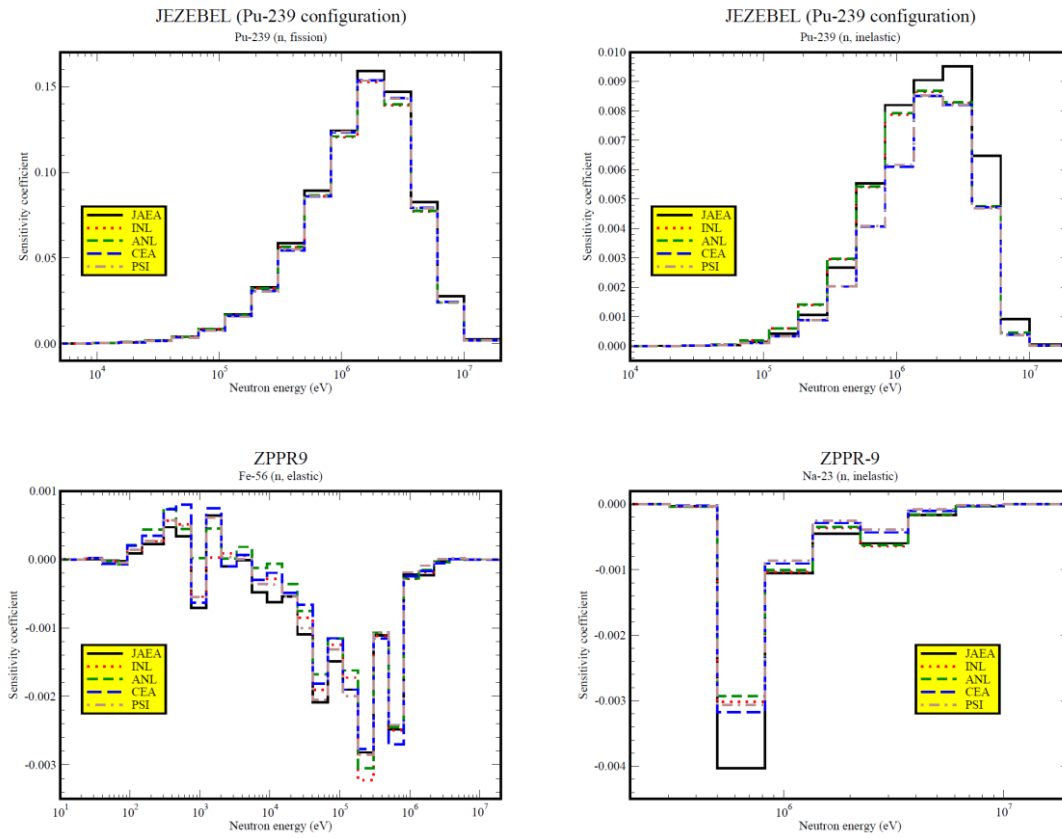


Fig. 5.1: Some consistent sensitivity profiles of F49/F25

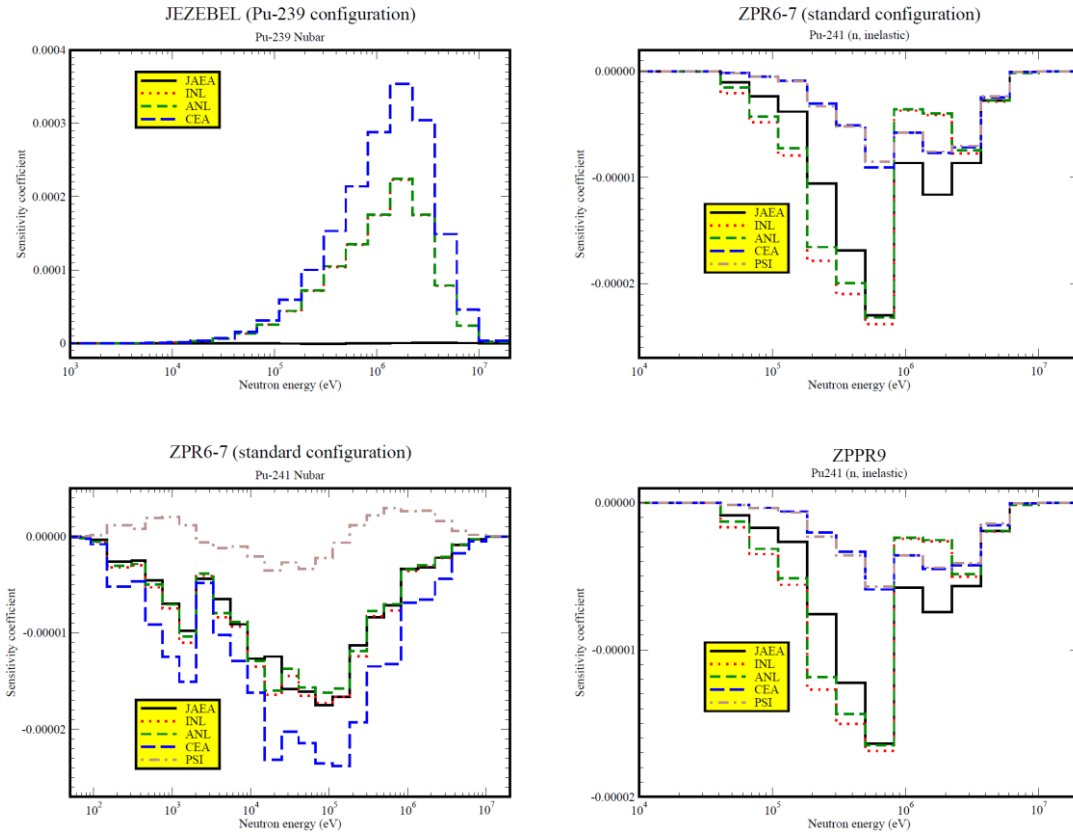


Fig. 5.2: Some discrepant sensitivity profiles of F49/F25

#### 4.2.3 F28/F25

As compared to F29/F25, a larger number of discrepant sensitivity profiles are noticed, which concern, in addition to  $\nu$ , inelastic scattering of actinides in general (see Fig. 6), and this for all four systems involved, namely JEZEBEL, FLATTOP, ZPR6-7 (standard configuration) and ZPPR9. No systematic trend can easily be identified to characterize differences between the various solutions.

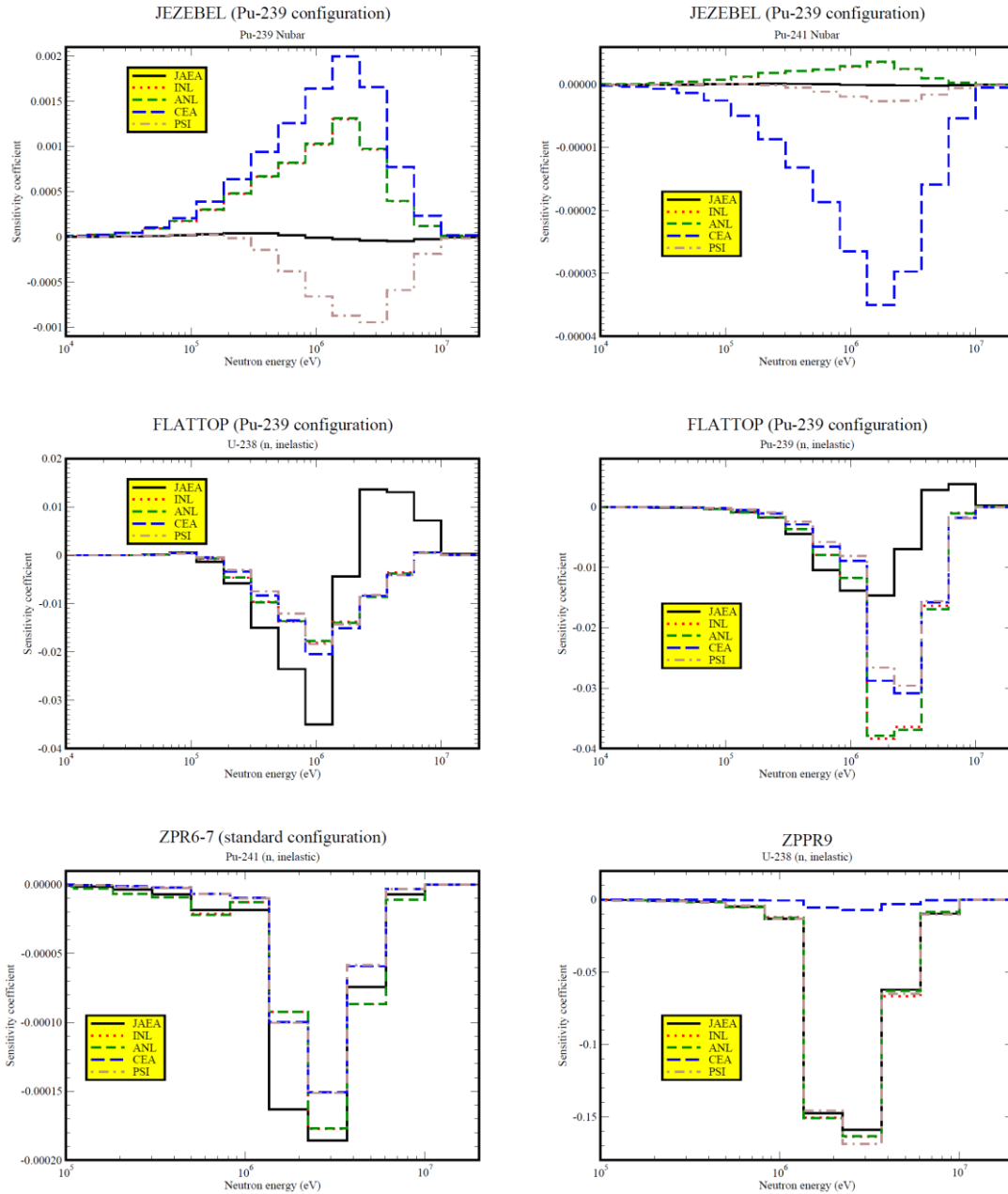


Fig. 6: Some discrepant sensitivity profiles of F28/F25

#### 4.2.4 F37/F25

The deviations, in this case, occur in a less systematic manner as compared to F49/F25 or F28/F25. Namely,  $\nu$  is of concern just in the case of JEZEBEL ( $^{239}\text{Pu}$  configuration), whereas for FLATTOP, the second system in which F37/F25 is involved, the sensitivity coefficients for elastic scattering of  $^{241}\text{Pu}$ , inelastic scattering of  $^{235}\text{U}$ ,  $^{240}\text{Pu}$ , and  $^{241}\text{Pu}$ , as well as  $^{240}\text{Pu}$  fission are those which deviate at most (see Fig. 7). One should however keep in mind that some of the differences pointed out indiscriminately in the study may refer to very small values likely unimportant for uncertainty calculations.

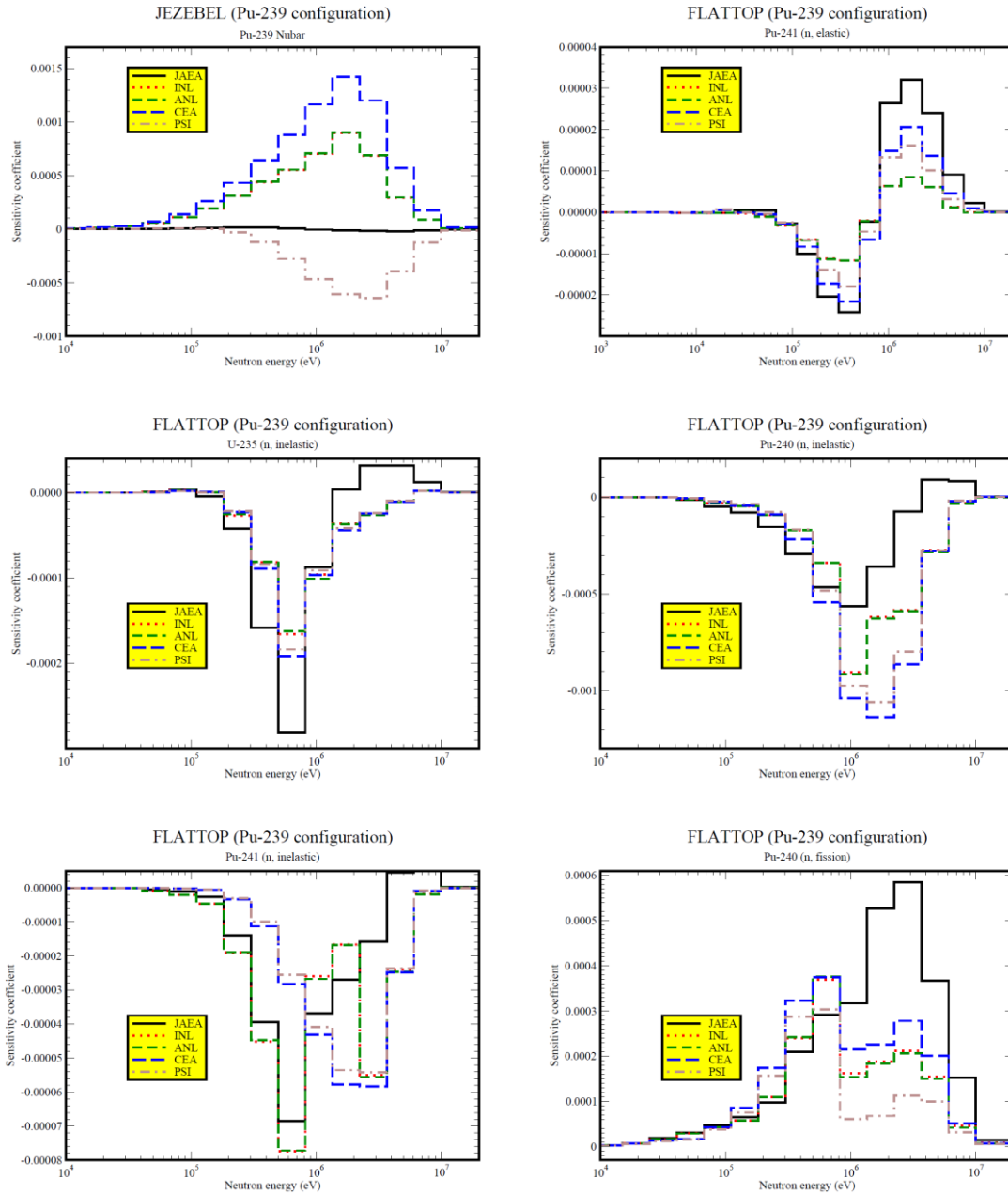
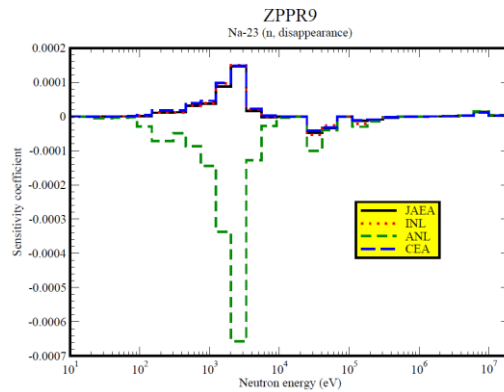
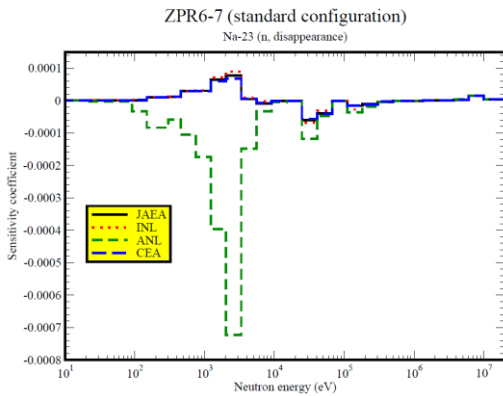
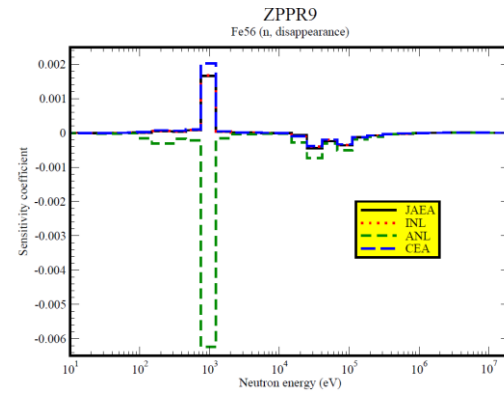
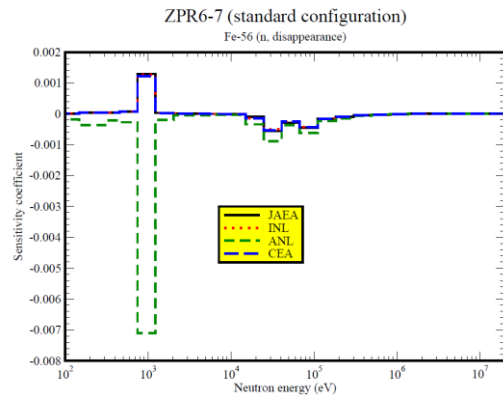
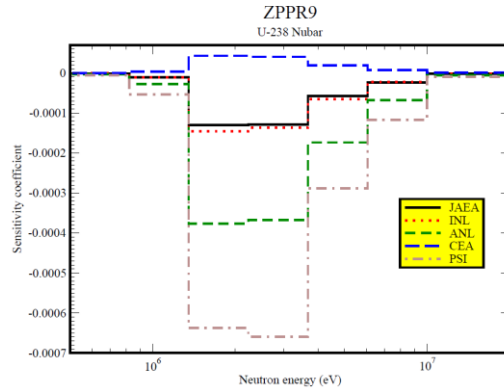
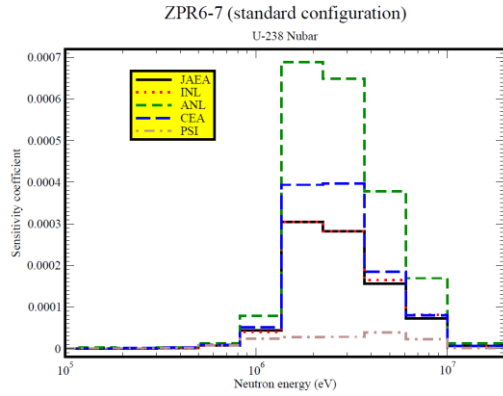


Fig. 7: Some discrepant sensitivity profiles of F37/F25

#### 4.2.5 C28/F25

For the two systems involved, namely ZPR6-7 (standard configuration) and ZPPR9, the main discrepancies of the sensitivity coefficients concern  $\nu$  for  $^{238}\text{U}$ , though the values are small, disappearance of sodium and structural materials, the PSI solution being not available for this lumped reaction, elastic scattering of  $^{58}\text{Ni}$  and inelastic scattering of  $^{56}\text{Fe}$  (see Fig. 8). As opposed to the other parameters studied, for these reactions there are large differences between the INL and ANL solutions which were both obtained with ERANOS in conjunction with ENDF/B-VII.0 data (compare red with green curves), the former being much closer to the CEA solution based upon the use of JEFF-3 data (blue curves). In the case of disappearance, the ANL sensitivity coefficient is the only one which is negative in the lower keV range, whereas  $^{58}\text{Ni}$  elastic shows

an opposite trend, but this near 20keV. Finally, for  $^{56}\text{Fe}$  inelastic, the ANL value is about twice the average value of the other solutions around 1MeV.



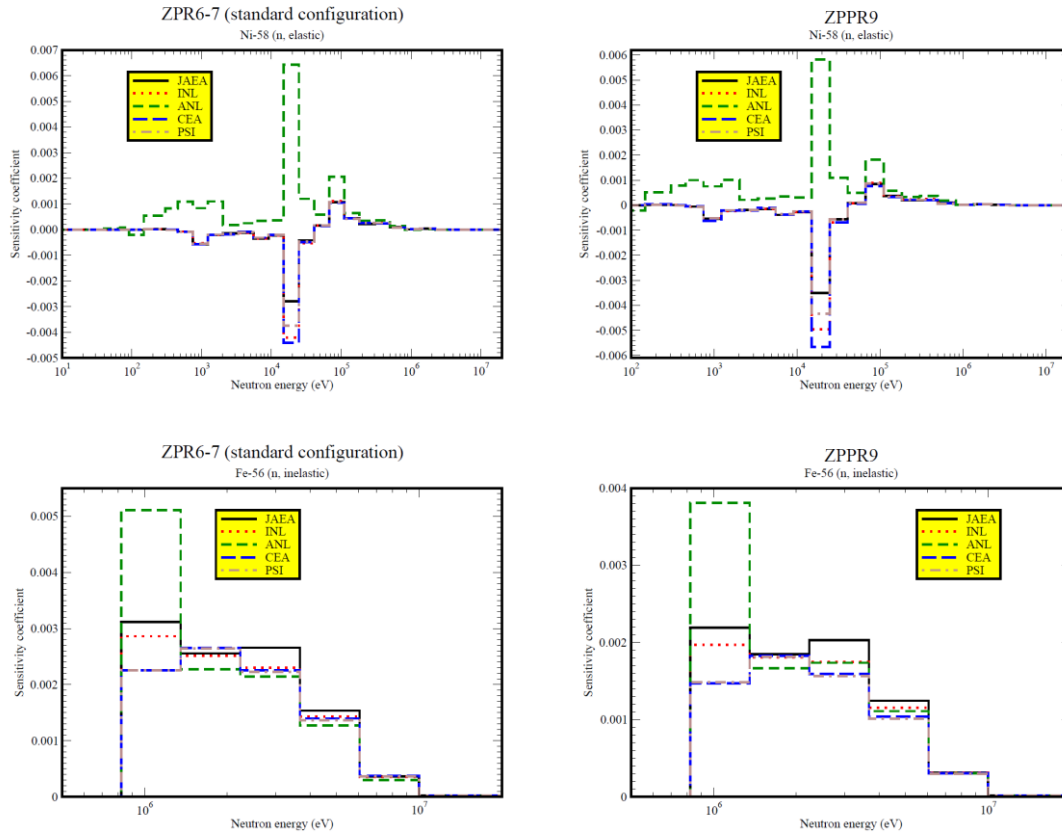


Fig. 8: Discrepant sensitivity profiles of C28/F25

#### 4.2.6 Na void (Step 3) and Na void (Step 5)

For the majority of the sensitivity profiles, consistence is shown among the participants (JAEA, INL, ANL, CEA, and PSI) providing for ZPPR9 deterministic solutions based upon the EGPT methodology (see Fig. 9.1). Larger differences are only observed for <sup>240</sup>Pu (fission,  $\nu$ , elastic and inelastic scattering), as well as <sup>241</sup>Pu elastic and inelastic scattering just for Na void (Step 5), see Fig. 9.2, in which case, however, the values are very small.

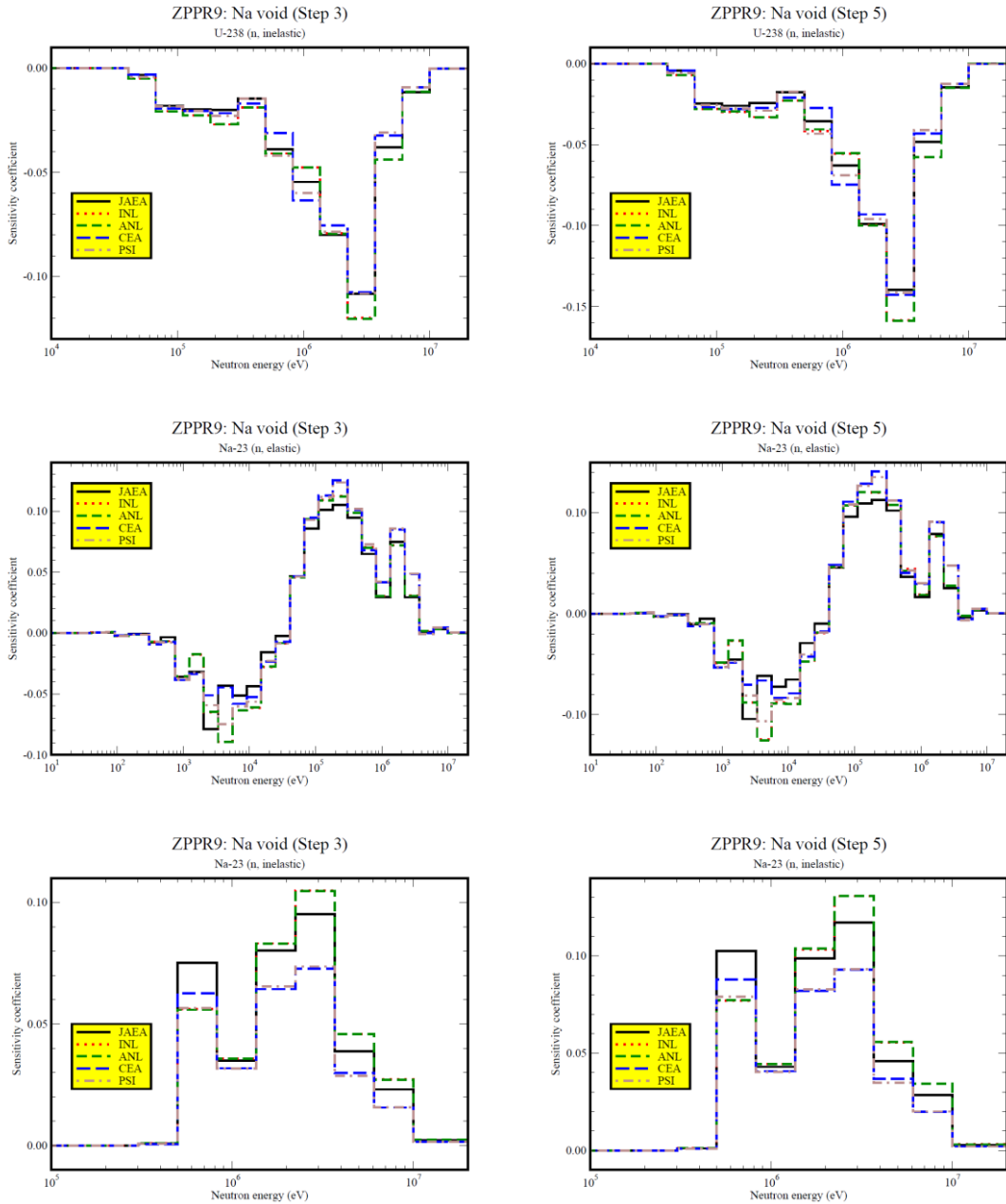


Fig. 9.1: Some consistent sensitivity profiles for the void effect

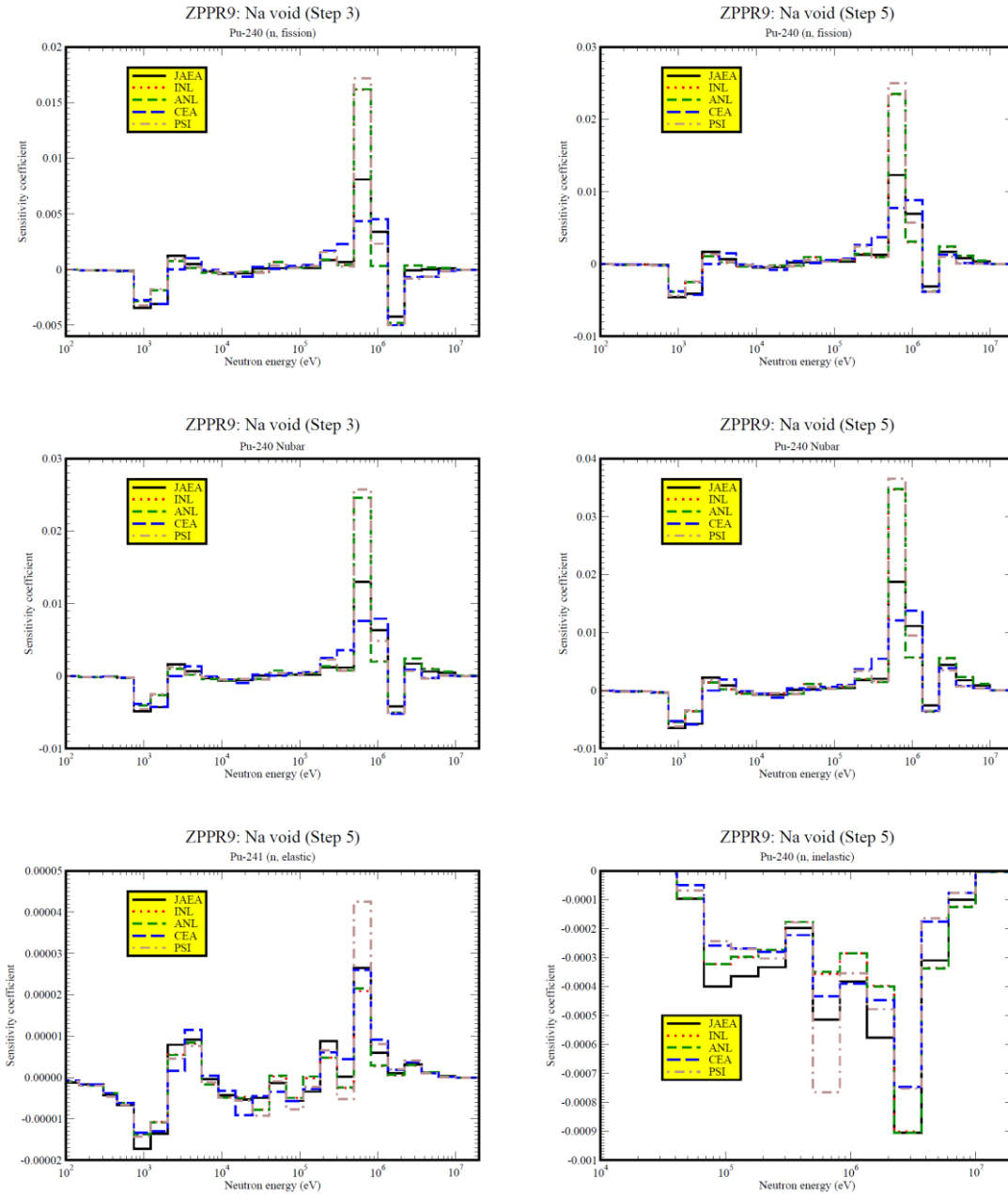


Fig. 9.2: Discrepant sensitivity profiles for the void effect

For  $\nu$  and the fission reactions, the main discrepancies are observed near 1MeV. While the largest value of the sensitivity coefficients is consistently reached by the INL, ANL and PSI solutions (respectively red, green and brown curves), the smallest value, which is about one third of the largest value, is given by the CEA calculation (blue curve).

### 4.3 Conclusions

Sensitivity coefficients of the effective multiplication factor have been obtained by JAEA, INL, ANL, CEA, PSI, IJS and KAERI for all benchmark systems, namely JEZEBEL, ( $^{239}\text{Pu}$  and  $^{240}\text{Pu}$  configurations),

FLATTOP ( $^{239}\text{Pu}$  configuration), ZPR6-7 (standard configuration and high  $^{240}\text{Pu}$  content), ZPPR9 and JOYO, and these from independent deterministic solutions based upon the standard perturbation theory methodology [4]. In addition, IRSN has used for all these configurations a stochastic methodology to correspondingly derive total sensitivity coefficients (see Section 4.2.1).

Except for the KAERI solution, overall consistence of the results has been shown among the participants. The major discrepancies affect FLATTOP, especially scattering reactions (of the actinides). Besides FLATTOP, a Pu core surrounded by a U reflector, the use of total sensitivity coefficients seems to have a non negligible impact particularly as regards scattering reactions in general for the sodium-cooled systems ZPR6-7, ZPPR9, and JOYO which are more sensitive to mutual resonance shielding effects.

Sensitivity coefficients of measured reaction rates at the core center relative to  $^{235}\text{U}$  fission, i.e. F49/F25, F28/F25, F37/F25, and C28/F25 (F49 standing for  $^{239}\text{Pu}$  fission, F28 for  $^{238}\text{U}$  fission, F37 for  $^{237}\text{Np}$  fission, and C28 for  $^{238}\text{U}$  capture) have been derived by JAEA, INL, ANL, CEA, and PSI on the basis of generalized perturbation theory [4]. In several cases overall consistence of the results has also been shown among the participants. For the fission reactions, however, larger discrepancies affect  $\nu$  and inelastic scattering for actinides in general (see Sections 4.2.2-4.2.4); for C28/F25 (see Section 4.2.5), the sensitivity coefficients of disappearance for structural materials in general are deviating, as well as those for  $^{58}\text{Ni}$  elastic and  $^{56}\text{Fe}$  inelastic scattering (see Fig. 8).

Finally, equivalent generalized perturbation theory [4] has been used for determining the sensitivity coefficients of measured void reactivity effects in ZPPR9, i.e. Na void (Step 3) and Na void (Step 5), showing overall consistence of the various results except for  $^{240}\text{Pu}$  in general (see Section 4.2.6, Fig. 8).

It clearly will be of interest to investigate how far these discrepancies will be reduced after adjustment.

## References

- [1] "SCALE, a modular code system for performing standardized computer analyses for licensing evaluations. Version 6: Report available from the Radiation Safety Information Computational Center at Oak Ridge National Laboratory as CCC-750. ORNL/TM-2005/39, Oak Ridge National Laboratory (2009)
- [2] D. Rochmann, "Adjusting  $^{235}\text{U}$ ,  $^{238}\text{U}$  and  $^{239}\text{Pu}$  nuclear data with a Monte Carlo technique", private communication, SG33 group meeting (May 22-23, 2012)
- [3] H. Wu, "Progress on nuclear data adjustment", private communication, SG33 group meeting (May 22-23, 2012)
- [4] J. Tommasi, "ERANOS User's manual- Applications of perturbation theory with finite difference diffusion and  $S_n$  transport flux solvers", CEA report, SPRC/LEPh 07-003 (2007)
- [5] M. Ishikawa, "Recent application of nuclear data to fast reactor core analysis and design in Japan", International Conference on Nuclear Data for Science and Technology (2005)
- [6] G. Rimpault et al., "The ERANOS data and code system for fast reactor neutronic analyses", Proceedings of the International Conference on the New Frontier of Nuclear Technology: Reactor Physics, Safety and High-Performance Computing (PHYSOR 2002), Seoul, Korea (2002)
- [7] R. E. Alcouffe et al., "DANTSYS: A Diffusion Accelerated Neutral Particle Transport Code System", Los Alamos National Laboratory report, LA-12969-M (1995)
- [8] I. Kodeli, "Multidimensional Deterministic Nuclear Data Sensitivity and Uncertainty Code System: Method and Application", Nuclear Science and Engineering, 138, (2001), pp. 45-66

[9] “TSUNAMI-3D: Control module for three-dimensional cross-section sensitivity and uncertainty analysis for criticality (SCALE, VOL I, Sect. C9). Technical Report ORNL/TM-2005/39, Oak Ridge National Laboratory (2009)