

# Generation of fission yield covariances to correct discrepancies in the JEFF fission yield library

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STUDIECENTRUM VOOR KERNENERGIE  
CENTRE D'ETUDE DE L'ENERGIE NUCLEAIRE

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- ENDF-6-format libraries (JEFF, ENDF/B, ...):

- Independent Fission Yields [Y]

- Cumulative Fission Yields [C]

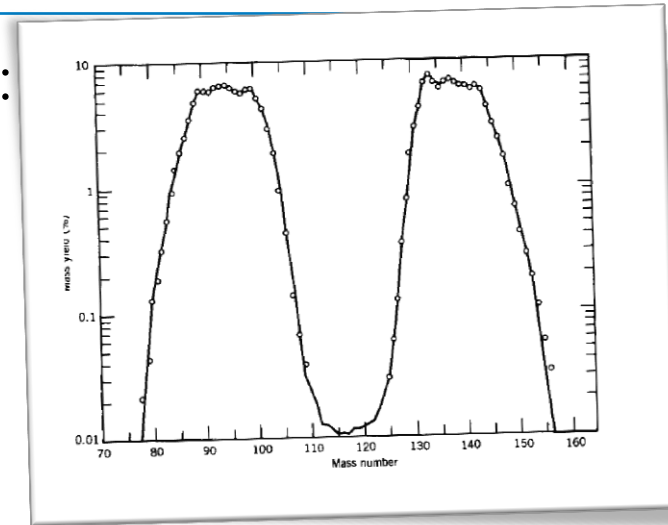
- Best estimate

- Uncertainties

**No covariance  
matrix**

- ENDF-6-format libraries (JEFF, ENDF/B, ...):

- Independent Fission Yields [Y]
- Cumulative Fission Yields [C]
  - Best estimate
  - Uncertainties



- Physical constraints:

**Charge conservation**

$$\sum_i Z_i Y_i = Z_{CN} - Z_{LCP}$$

**Mass conservation**

$$\sum_i A_i Y_i = A_{CN} - \bar{\nu}_p(E) - A_{LCP}$$

**Individual charge conservation**

$$\sum_{Z=Z_1} Y = \sum_{Z=Z_2} Y \quad Z_1 + Z_2 = Z_{CN}$$

**Number of fission products**

$$\sum_i Y_i = 2$$

**Asymmetry**

$$\sum_{A_i > \frac{A_{CN} - \bar{\nu}_p(E)}{2}} Y(A_i) = 1$$

- ENDF-6-format libraries (JEF

- Independent Fission Yields
- Cumulative Fission Yields
  - Best estimate
  - Uncertainties

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**Individual charge conservation**

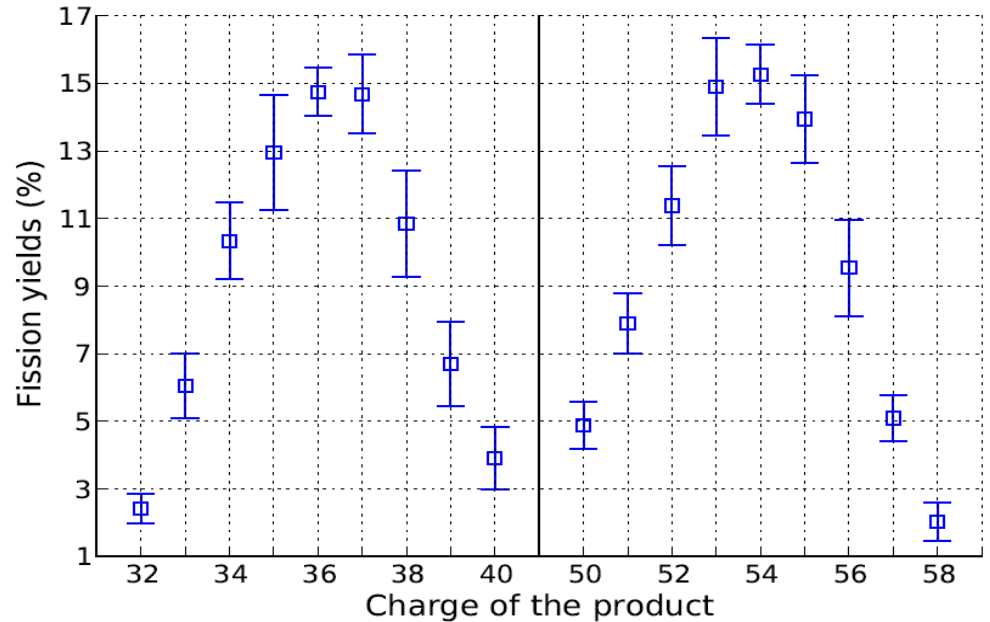
$$\sum_{Z=Z_1} Y = \sum_{Z=Z_2} Y \quad Z_1 + Z_2 = Z_{CN}$$

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- ENDF-6-format libraries (JEFF, ENDF/B, ...):

- Independent Fission Yields  $[Y]$

- Cumulative Fission Yields  $[C]$

- Best estimate

- Uncertainties

- Chain yields  $[Ch]$

- Best estimate and uncertainties available in the literature

(but not in the ENDF-6 library) • **ENDF-349, Evaluation and Compilation of Fission Product Yields (1993)**

• **IAEA, Handbook of Nuclear Data for Safeguards (2008)**

**Independent to Cumulative**

$$QY = C$$

**Independent to Chain**

$$\sum_{A_i=A^*} Y(A_i) = Ch(A^*)$$

# Need for correlations

- Best-estimate values comply with the constraints
- Uncertainty data show discrepancies ( $S^T V S$ )
- Example:
  - Calculation of burnup indicators

$$N_{Nd^{148}}(t) \approx \frac{\Sigma_f \phi C_{Nd^{148}}}{\sigma_c^{Nd^{148}} \phi} \left( 1 - e^{-\sigma_c^{Nd^{148}} \phi t} \right)$$

- Uncertainty on  $^{148}\text{Nd}$  cumulative fission yield

$$QY = C \quad \longrightarrow \quad V_C = Q^t V_Y Q$$

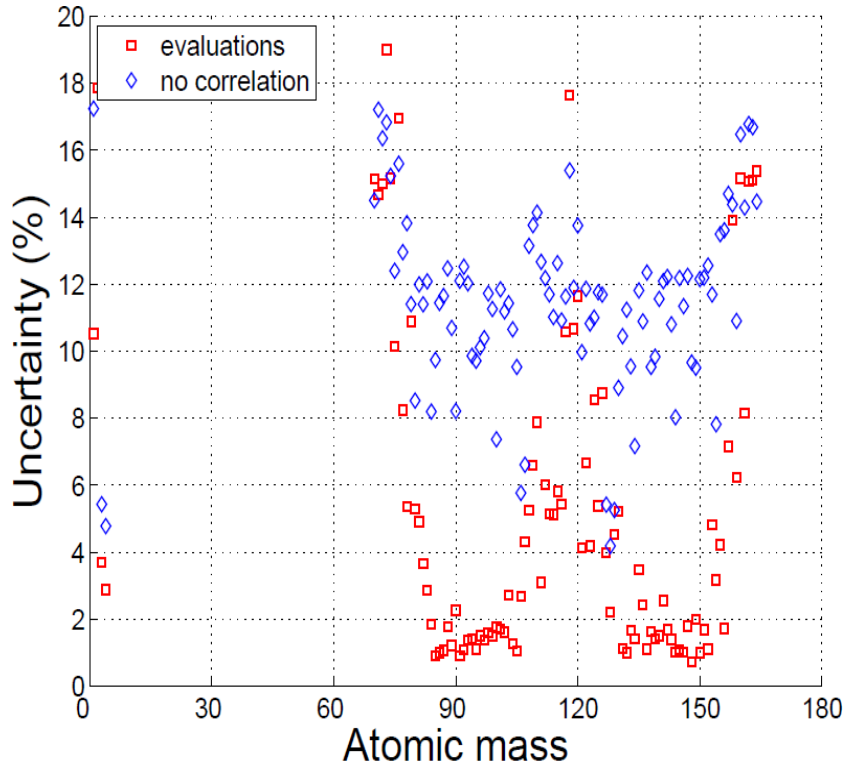


	JEFF-3.1.1	ENDF/B-VII.1
<b>Library</b>	0.7%	0.35%
<b>Calculated</b>	9.67%	21.42%

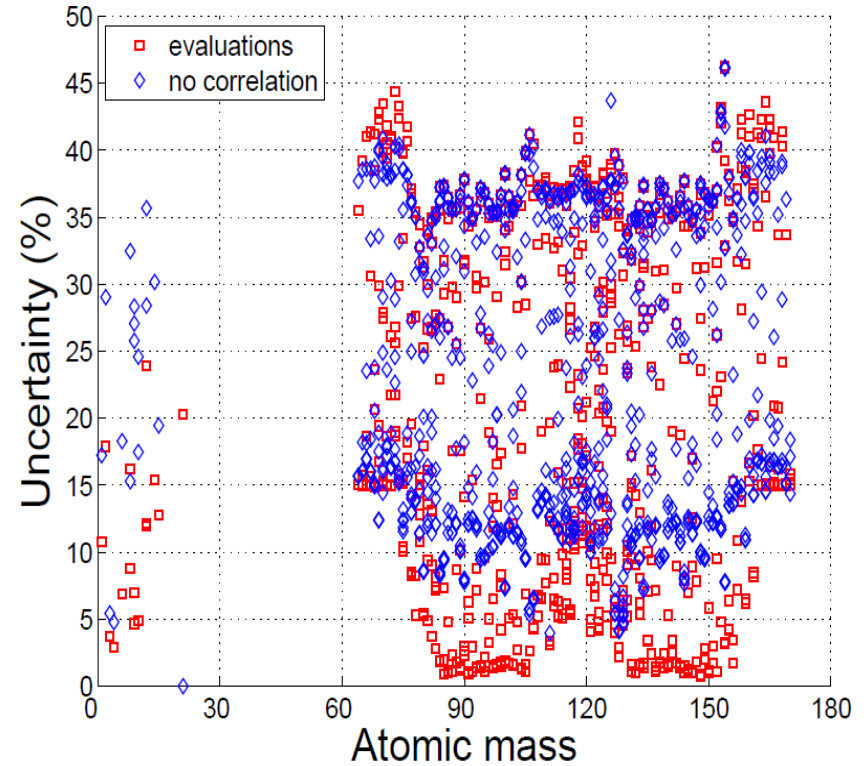
- Inconsistency in the moment propagation formula

# Discrepancy in the library (U235-th)

## ● Chain yields



## ● Cumulative yields

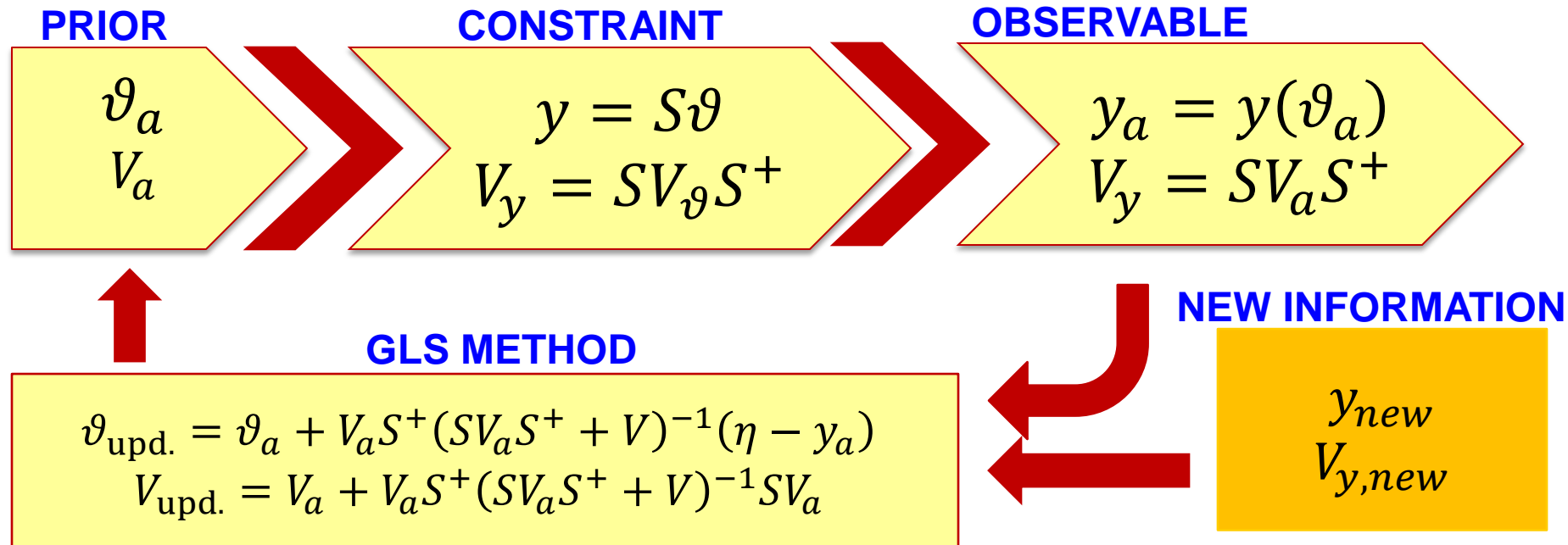


# Covariance matrix generation

- Generalised least square approach

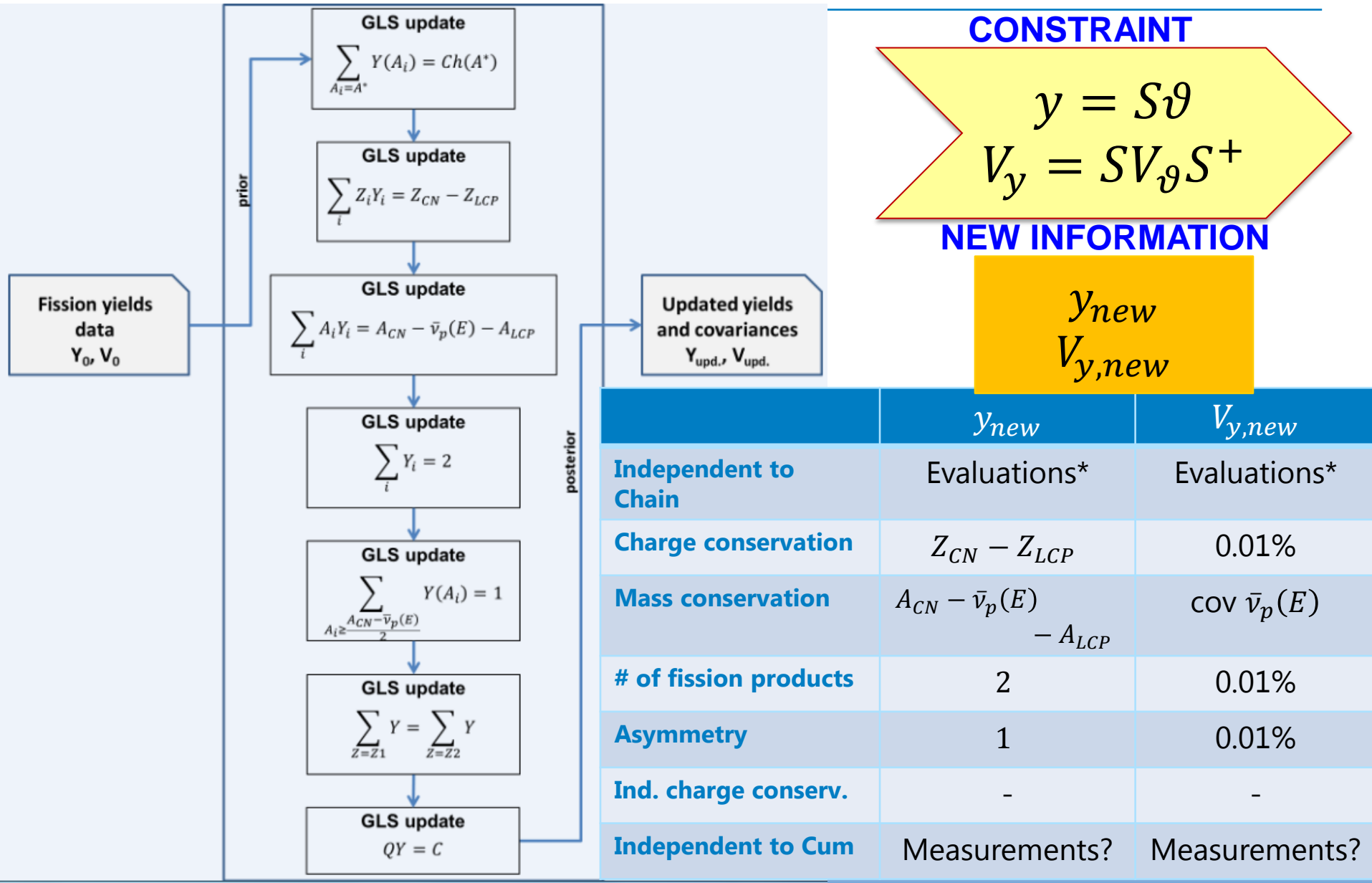
$$\chi^2 = (\vartheta - \vartheta_a)^+ V_a^{-1} (\vartheta - \vartheta_a) + (\eta - y)^+ V^{-1} (\eta - y) = \text{minimum}$$

- Updating procedure



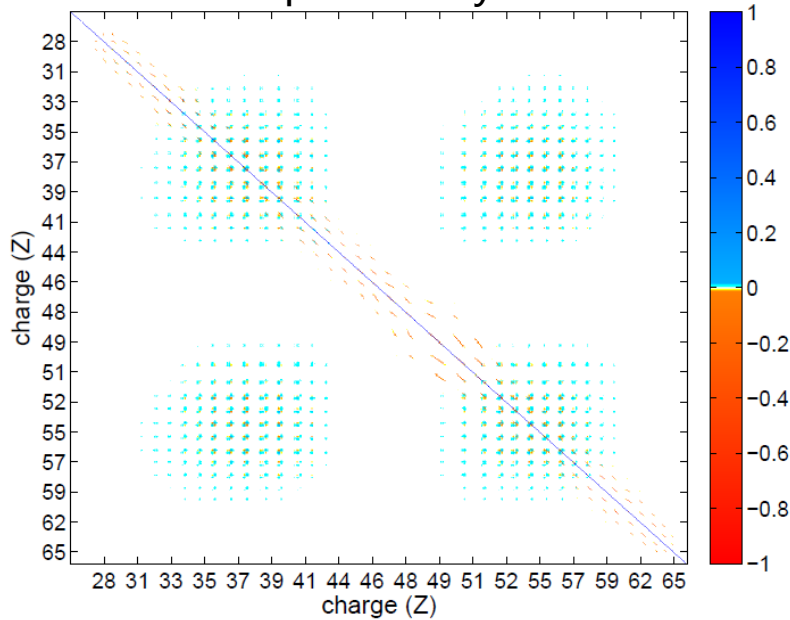
\*IAEA, Handbook of Nuclear Data for Safeguards (2008)

# Iterative updating process

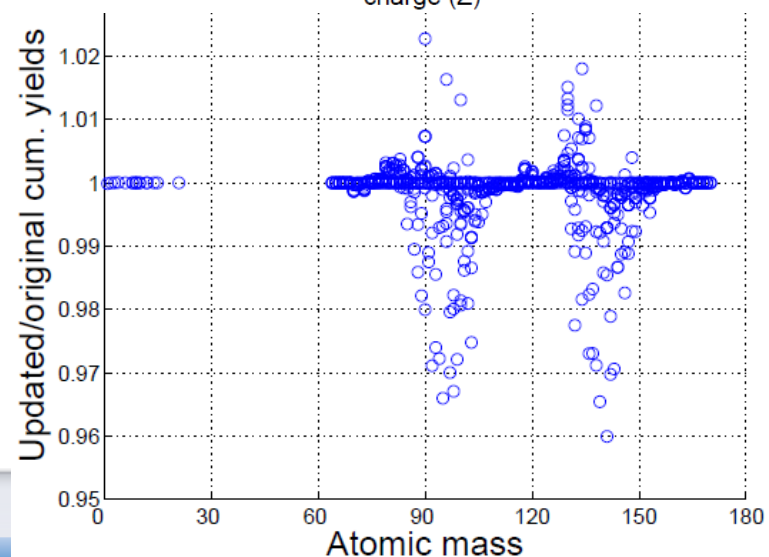
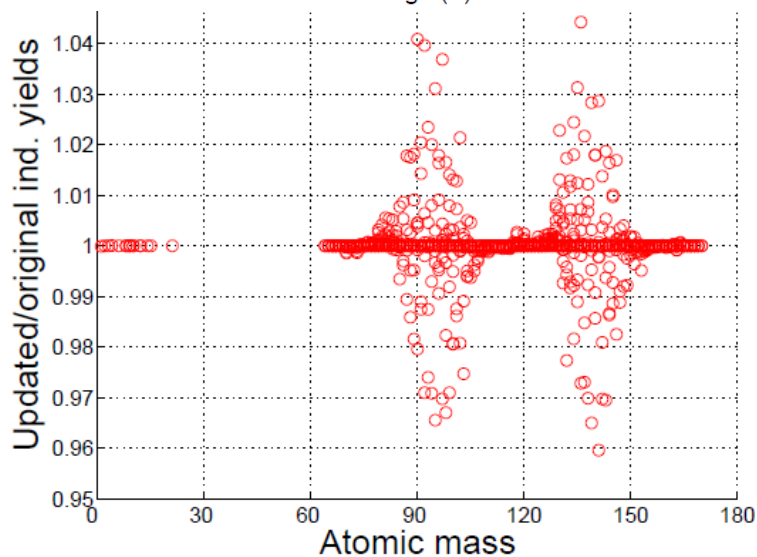
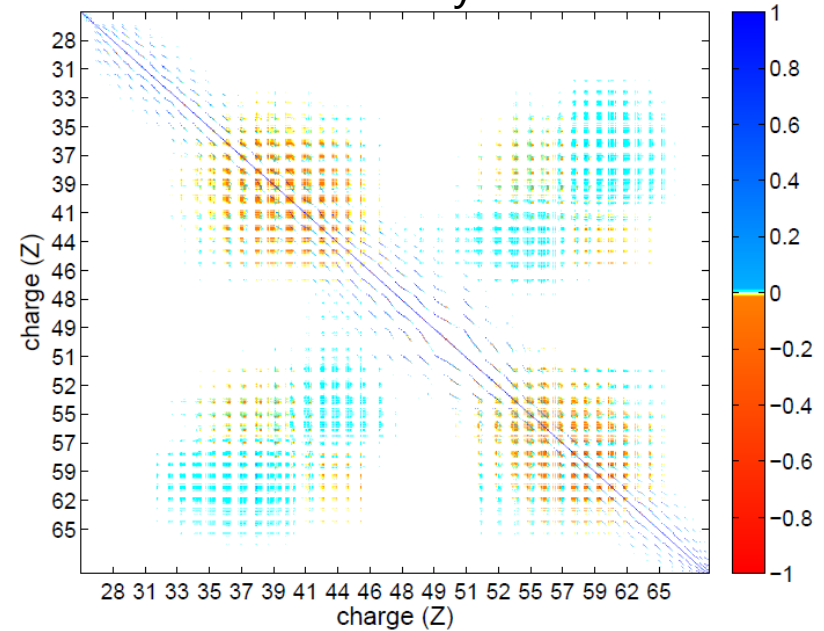


# Example of correlation matrix (U235-th)

Independent yields

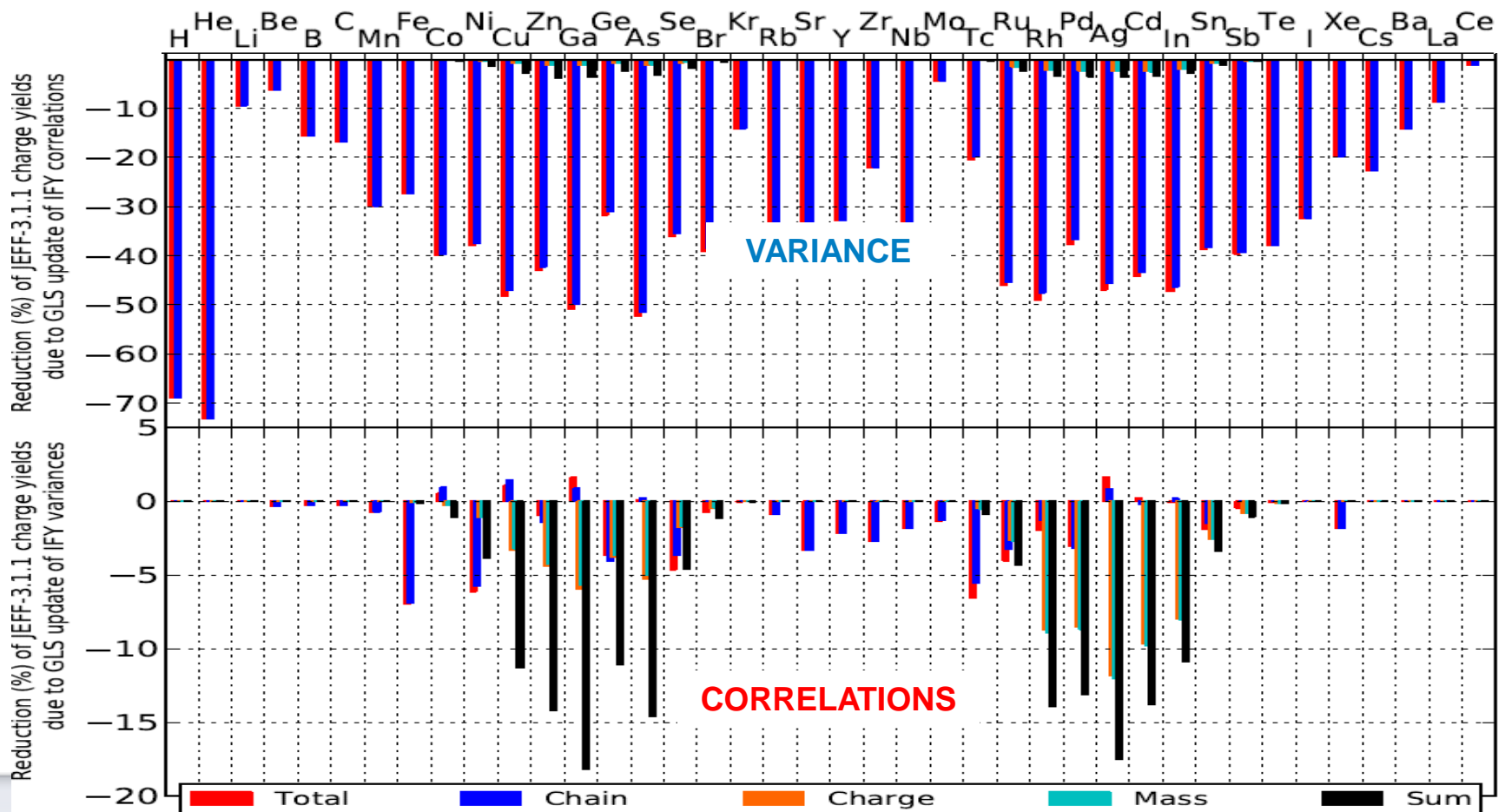


Cumulative yields



# Double effect of the updating process

$$\text{Charge yields} = \sum_{Z_i=Z^*} Y(Z_i)$$



# Need for correlations

- Example:
  - Calculation of burnup indicators

$$N_{Nd^{148}}(t) \approx \frac{\Sigma_f \phi C_{Nd^{148}}}{\sigma_c^{Nd^{148}} \phi} \left( 1 - e^{-\sigma_c^{Nd^{148}} \phi t} \right)$$

- We introduced correlations amongst the independent fission yields
- Uncertainty on <sup>148</sup>Nd cumulative fission yield

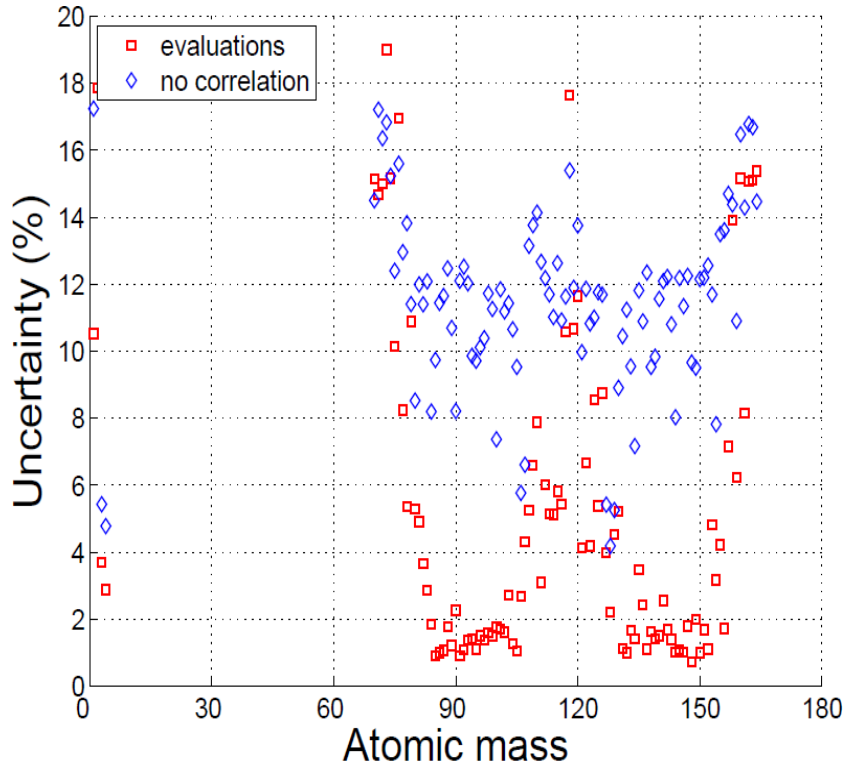
$$QY = C \quad \longrightarrow \quad V_C = Q^t V_Y Q \quad \text{WITH CORRELATIONS}$$

	JEFF-3.1.1	ENDF/B-VII.1
<b>Library</b>	0.7%	0.35%
<b>Calculated</b>	9.67%	21.42%
<b>Updated</b>	1.01%	0.35%

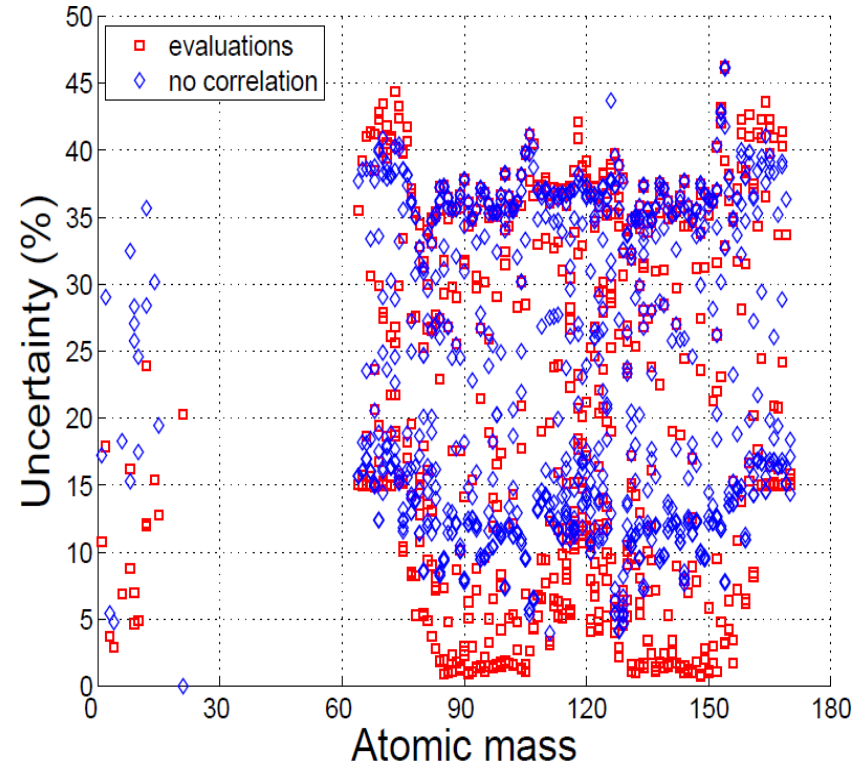
- Now the moment propagation formula is consistent

# Discrepancy in the library (U235-th)

## ● Chain yields

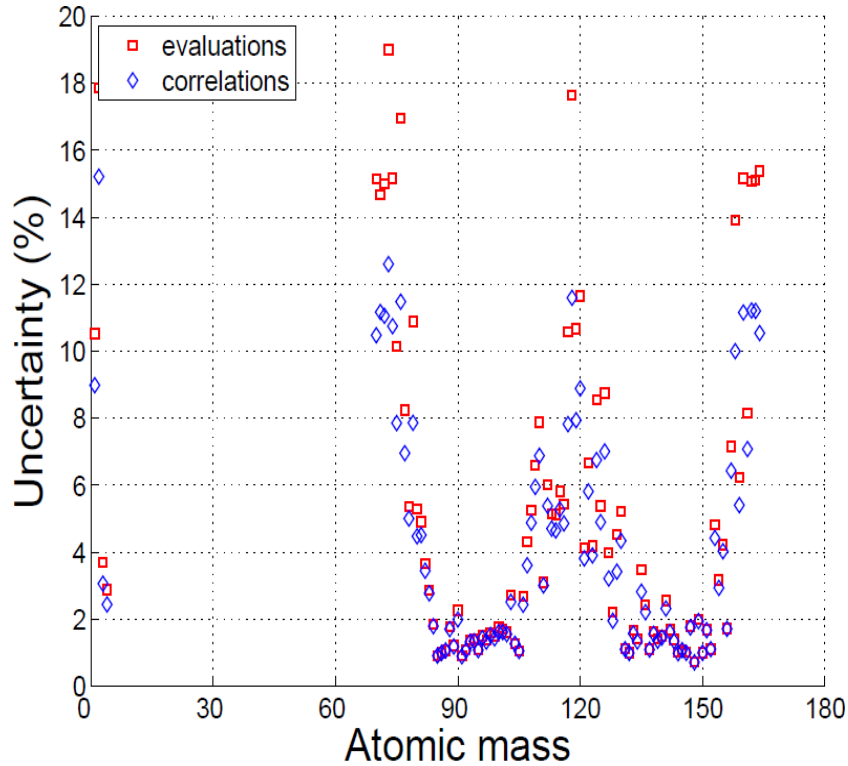


## ● Cumulative yields

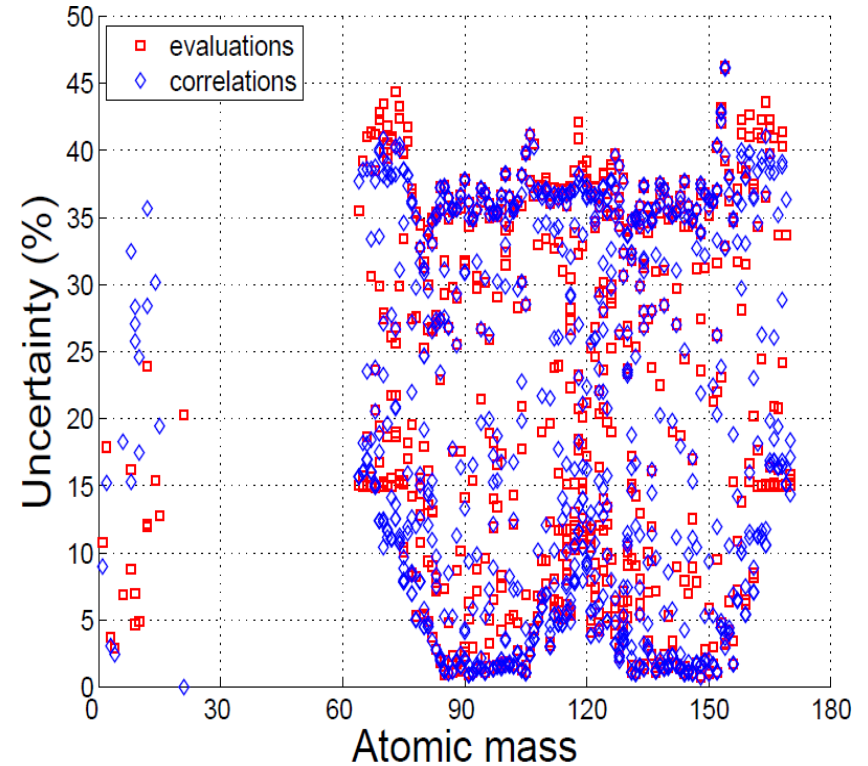


# Discrepancy in the library (U235-th)

## ● Chain yields



## ● Cumulative yields

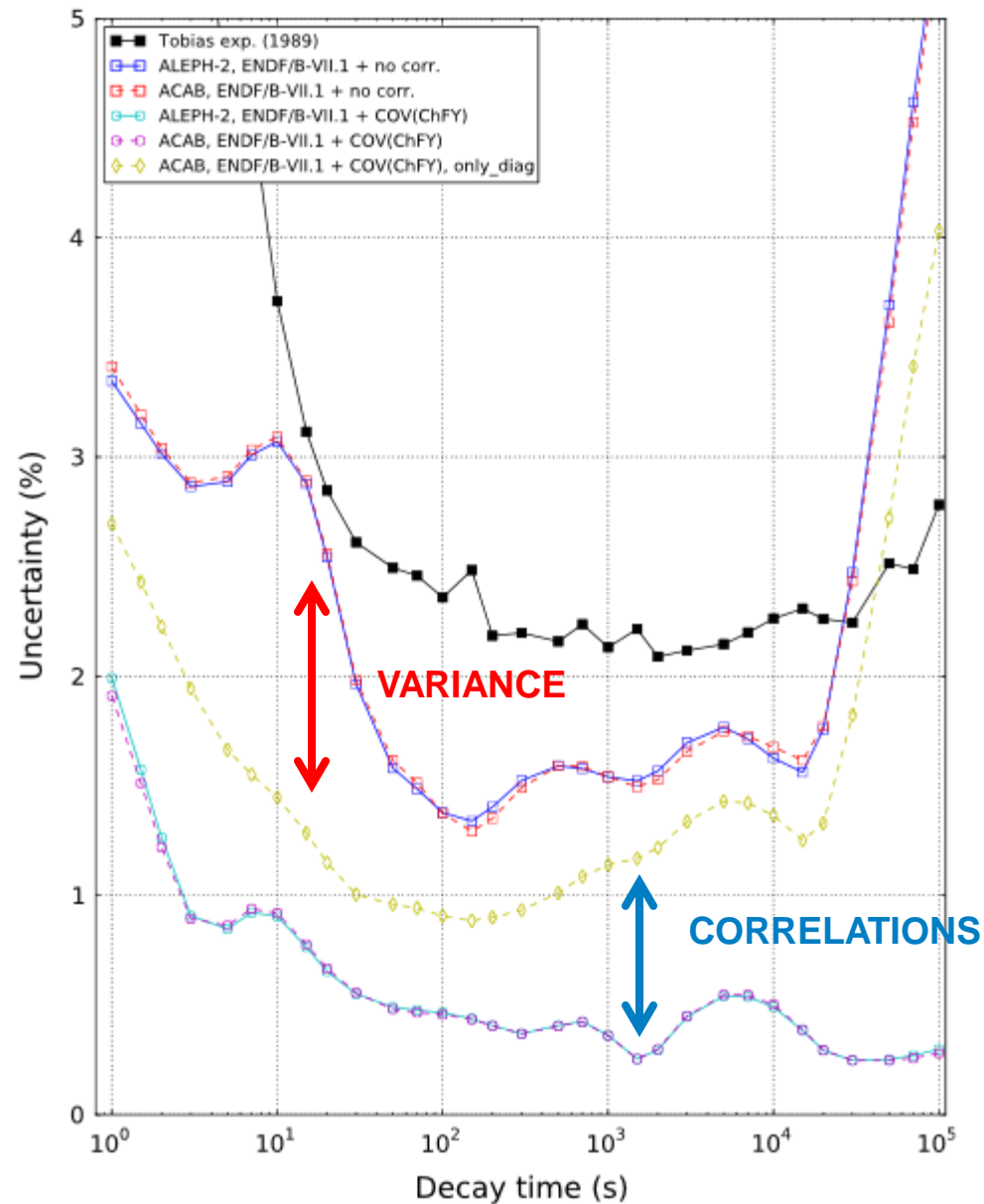


# Summary

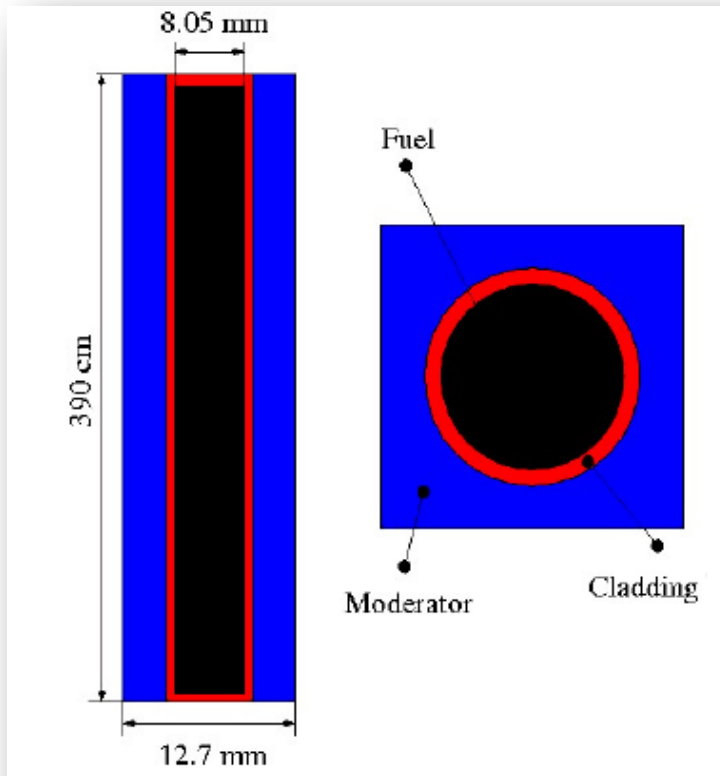
- Generation of correlations between fission yields
- Covariances represent physical constraints and evaluated uncertainties
- Correlations solve the discrepancies in the libraries
- We applied our covariance update procedure to **14 systems of the JEFF-3.1.1 library**:
  - Th-232 fast, 14-MeV
  - U-233 thermal, fast, 14-MeV
  - U-235 thermal, fast, 14-MeV
  - U-238 fast, 14-MeV
  - Pu-239 thermal, fast
  - Pu-241 thermal, fast
- Attempt to create inter-energy correlations using  $\bar{\nu}$  correlations
- **New ENDF-like format to store covariances** (mix MF=8, MF=33) **MF=38 MT=454**

```
9.02320E+4 2.30045E+2      2      0      0      0348638454
4.00000E+5 0.00000E+0      1      5     348194     833348638454
1.0010E+03 0.0000E+00 1.0020E+03 0.0000E+00 1.0030E+03 0.0000E+00348638454
2.0040E+03 0.0000E+00 2.5064E+04 0.0000E+00 2.5065E+04 0.0000E+00348638454
2.5066E+04 0.0000E+00 2.5067E+04 0.0000E+00 2.5068E+04 0.0000E+00348638454
2.5069E+04 0.0000E+00 2.6064E+04 0.0000E+00 2.6065E+04 0.0000E+00348638454
2.6066E+04 0.0000E+00 2.6067E+04 0.0000E+00 2.6068E+04 0.0000E+00348638454
2.6069E+04 0.0000E+00 2.6070E+04 0.0000E+00 2.6071E+04 0.0000E+00348638454
2.6072E+04 0.0000E+00 2.7065E+04 0.0000E+00 2.7066E+04 0.0000E+00348638454
2.7067E+04 0.0000E+00 2.7068E+04 0.0000E+00 2.7068E+04 1.0000E+00348638454
2.7069E+04 0.0000E+00 2.7070E+04 0.0000E+00 2.7070E+04 1.0000E+00348638454
```

# Impact on FPDH calculations



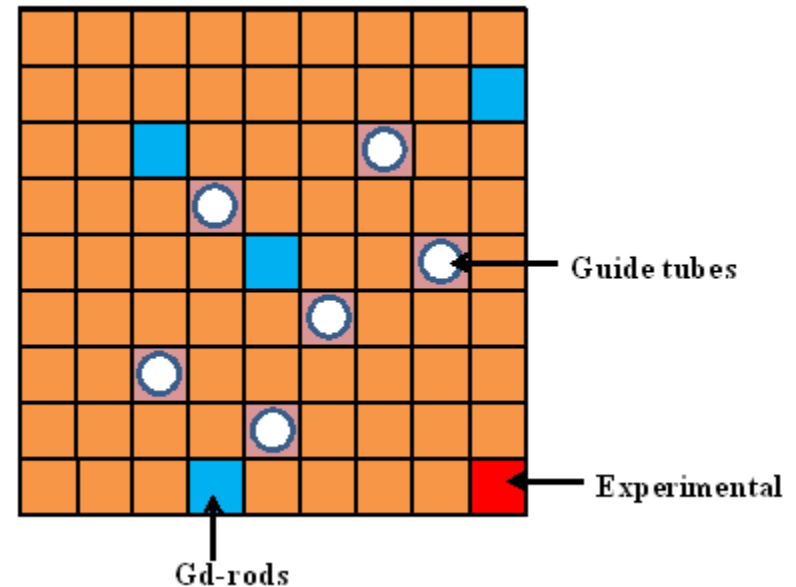
# REBUS singlepin model



REBUS: irradiation of samples ( $\text{UO}_2$  fuel elements among them) in commercial PWR at Neckarwestheim NPP (Germany)

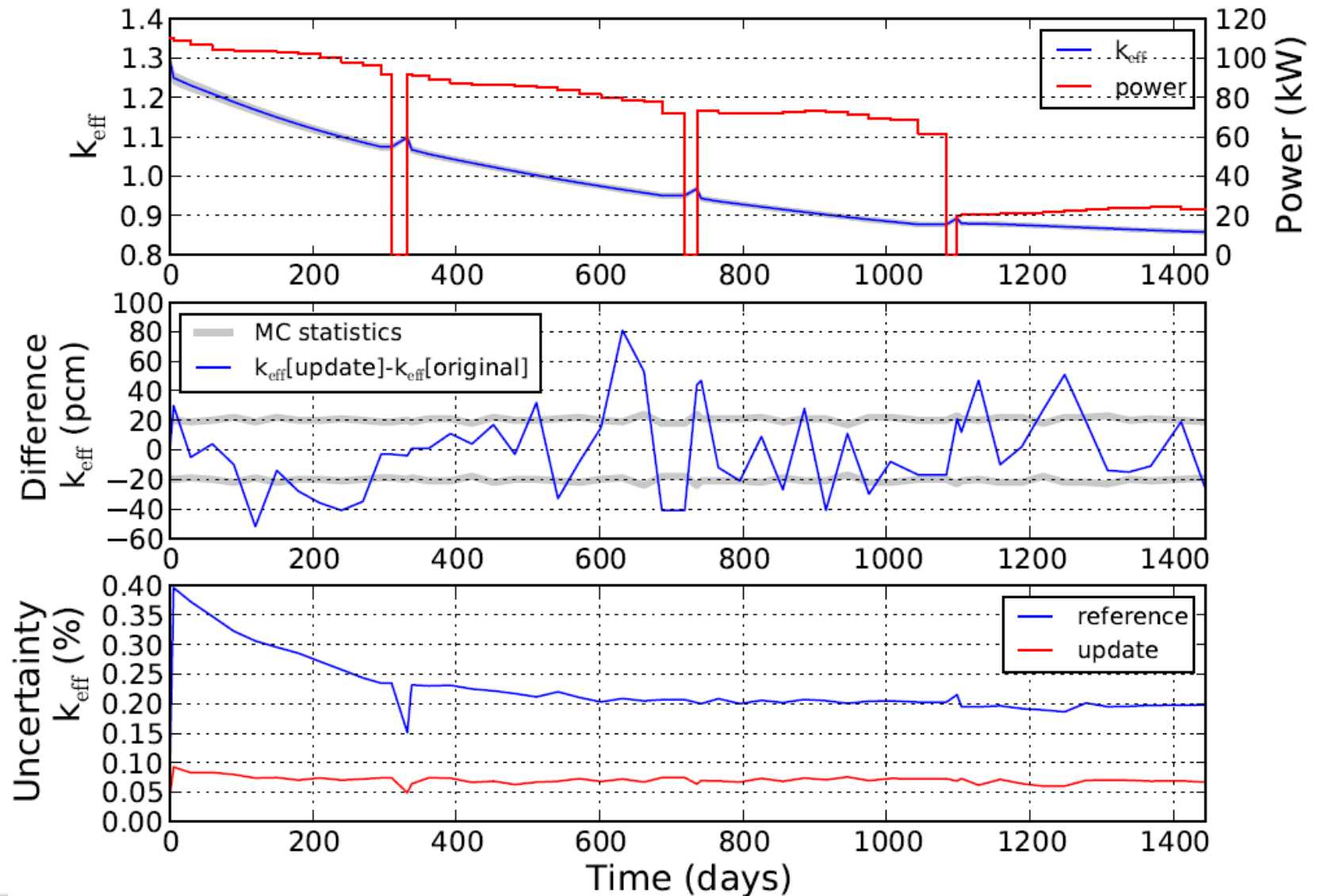
Gamma spectroscopy → burn-up profile

Radiochemical analysis → inventories of radionuclides

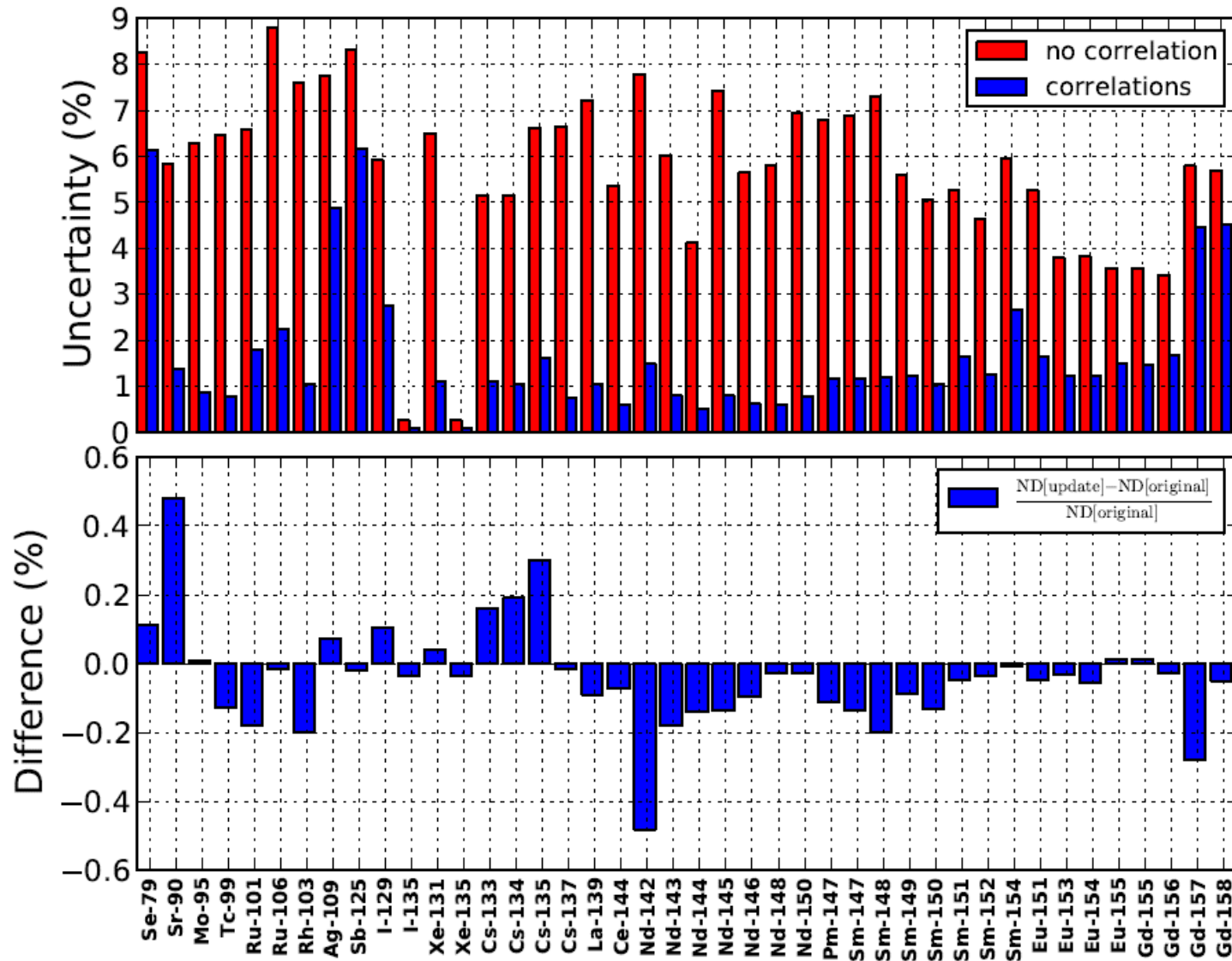


B. Lance. REBUS International Programme, REBUS-PWR Final Report. Ref. No./ File 0501320/221-1 (RE 2005/37). SCK·CEN, Belgium, 2006

# Impact on $k_{\text{eff}}$



# Impact on nuclide density uncertainties



# Conclusions

- Discrepancies in uncertainties
- Covariance generation by GLS method
- We generated covariance matrices for (almost) all the systems in JEFF-3.1.1
- Covariance matrices are stored in new format MF=38, MF=454
- Fission yield covariance matrices help reduce the response uncertainties to acceptable values

