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# A "Tiny" Adjustment of Nuclear Data and Associated Correlation Factor

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# Background

- According to the adjustment theory, posterior-correlations of nuclear data are inevitably generated if nuclear data evaluators consult C/E values of integral experiments, in order to adjust/calibrate a certain nuclear data such as a particular nuclide or reaction or energy region.

- However, this discussion is still qualitative, since we have not shown quantitative impacts on the posterior-correlations.

- We will try to show the impact on the posterior-correlations quantitively by using a very simple problem.

# Theory of Nuclear Data Adjustment

The nuclear data set after adjustment $\mathbf{T}'$ and covariance matrix $\mathbf{M}'$ are expressed as

$$\mathbf{T}' = \mathbf{T_0} + \mathbf{M}\mathbf{G}^{\mathbf{T}}\left(\mathbf{G}\mathbf{M}\mathbf{G}^{\mathbf{T}} + \mathbf{V_{em}}\right)^{-1}\left(\mathbf{R_e} - \mathbf{R_c}(\mathbf{T_0})\right)$$
$$\mathbf{M}' = \mathbf{M} - \mathbf{M}\mathbf{G}^{\mathbf{T}}\left(\mathbf{G}\mathbf{M}\mathbf{G}^{\mathbf{T}} + \mathbf{V_{em}}\right)^{-1}\mathbf{G}\mathbf{M}$$

The posterior correlations are independent of how much the nuclear data is adjusted!

where

- $\mathbf{T_0}$ : the nuclear data set before adjustment,
- $\mathbf{M}$ : the prior-covariance matrix,
- $\mathbf{G}$: the sensitivity of integral data with respect to nuclear data,
- $\mathbf{V_{em}}$ : the experimental and analytical uncertainty of integral data
- $\mathbf{R_e}$ : the measurement values of integral data,
- $\mathbf{R_c}(\mathbf{T_0})$ : the calculation values of integral data.

# Posterior-Correlation Factor

Let us consider a very simple problem such that two nuclear data are adjusted by using one integral data. In this case, we can explicitly denote as

$$\mathbf{M} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} = \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix},$$
$$\mathbf{G} = (g_1 \quad g_2),$$
$$\mathbf{V_{em}} = v_1.$$

The posterior correlation matrix is defined by

$$\mathbf{C}' = \begin{pmatrix} c'_{11} & c'_{12} \\ c'_{21} & c'_{22} \end{pmatrix}.$$

Then, we can explicitly write the posterior-correlation factor as

$$c'_{12} = \frac{-\sigma_1^2 g_1 \sigma_2^2 g_2}{\sqrt{D\sigma_1^2 - \sigma_1^4 g_1^2}\sqrt{D\sigma_2^2 - \sigma_2^4 g_2^2}} = c'_{21},$$

where

$$D = g_1 m_1 g_1 + g_2 m_2 g_2 + v_1.$$

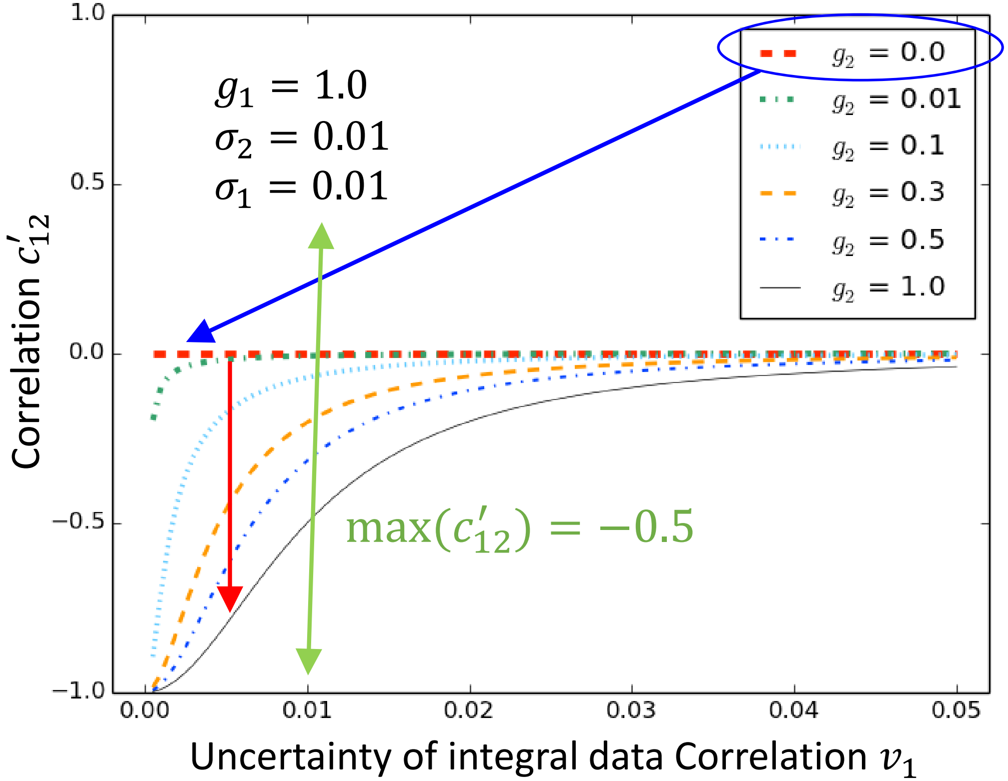
# Exercise with Simple Problem

- For example, let us consider a case where nu-bar of Pu-239 is slightly adjusted/calibrated to improve a Pu-239 criticality experiment.

- Posterior-correlation factors were calculated parametrically with the following condition:

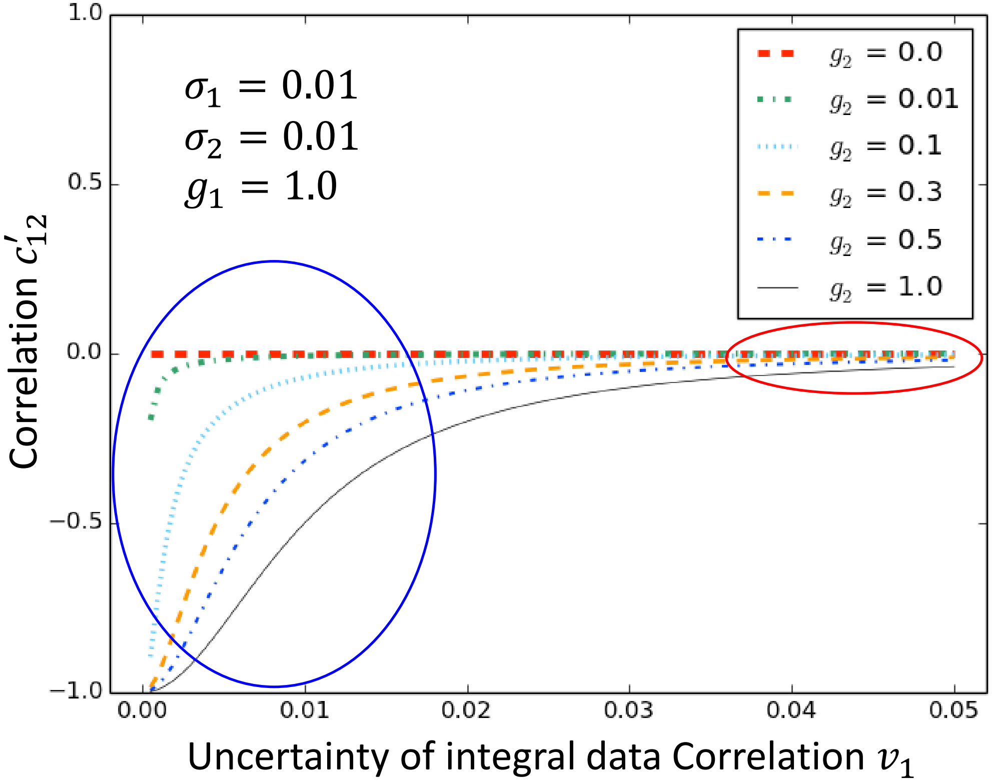
  - $\sigma_1$ (parameter): the prior-uncertainty of Pu-239 nu-bar
  - $g_1 = 1.0$ (100%, fixed): the sensitivity of Pu-239 nu-bar
  - $\sigma_2 = 0.01$ (1%, fixed): the prior-uncertainty of the other nuclear data
  - $g_2$ (parameter): the sensitivity of the other nuclear data (e.g., fission cross-section)
  - $v_1$ (parameter): the uncertainty of integral data

# Relationship with Uncertainty of Integral data (1/2)



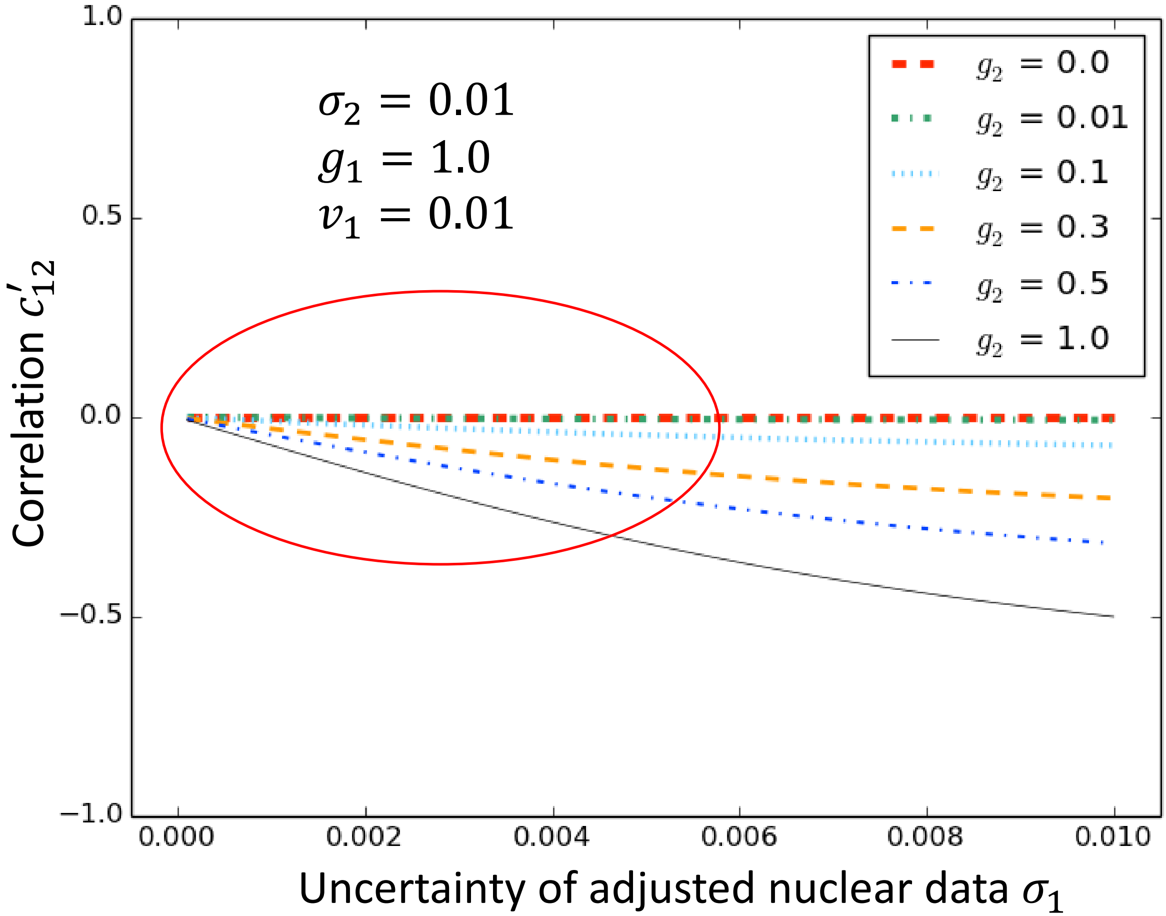

- When the sensitivity of other nuclear data ($g_2$) is zero, <u>no correlation is generated</u>.
- The larger the sensitivity of other nuclear data ($g_2$), <u>the stronger the posterior-correlation.</u>
- When the uncertainty of integral data ($v_1$) and the uncertainty of adjusted nuclear data ($\sigma_1$) are comparable, <u>the posterior-correlation factor becomes significant.</u>

# Relationship with Uncertainty of Integral data (2/2)




- When the uncertainty of integral data ($v_1$) is small, the posterior-correlation becomes strong.
- It is possible to ignore the posterior-correlation if the uncertainty of integral data ($v_1$) is extremely large.
  (However, evaluators would not refer to such bad integral data.)

# Relationship with Uncertainty of Adjusted Nuclear Data




- Even if the prior-uncertainty of the adjusted nuclear data ($\sigma_1$) is rather small, <u>the posterior-correlation could be significant</u> when the sensitivity of the other nuclear data (g2) is large.

# Summary

- We investigated the conditions where the posterior-correlation factor becomes significant or negligible in a simple adjustment case.
- The results are consistent with the qualitative trends expected from the adjustment equation.
- It would be possible that we estimate quantitatively the relationship between the posterior-correlation factor(s) and the other conditions, that is, nuclear data uncertainty, sensitivity and integral data uncertainty in a degree.