ECIS96*

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Abstract

Some improvements in `ECIS88` like the use of expansion of the potentials in term of Bessel functions lead to `ECIS94` and to write the **Notes on** `ECIS94` before I had to retire on the 1/9/1994. However, on the suggestion of Arjan Koning, the code was modified to deal with continuum for compound nucleus calculations. This point, existing in `ABAREX` was not in `ANLECIS`. Now, `ECIS95` should do the same kind of computation as `ABAREX`, but in the Coupled Channel formalism. Further work has been done this year by introducing energy dependent potentials defined by Mahaux' dispersion relations. This required more changes than the previous step and gives `ECIS96`. Some other additions should be done for experiments with exotic beams. Such beams are usually produced by extraction at 0 degrees after a reaction, which produces tensor polarisations very large for exotic nuclei with high spins. There are tensor polarisation effects on the total scattering cross-sections and on the total cross-section, even if they are generally too small to be measured.

1 Introduction.

The name `ECIS` ("*Equations Couplées en Itérations Séquentielles*") was introduced for an attempt to solve Coupled Channels problems with less computation [1]. Its principle is to obtain the few solutions needed in the application without handling all the solutions of the system of equations. The main application at that time was deformed spin-orbit interaction [2], which introduces first derivatives in the non diagonal part of the system of equations [3]. The presence of these first derivatives need numerical method far more lengthy than the usual way of solving Coupled Channels equations whereas the time needed by `ECIS` does not increase significantly. Of course, the usual method is also present in the code. The last version at this stage was `ECIS75`.

Application to heavy ions scattering needs long range integration because the Coulomb form factor for a multipole L decreases as r^{-L-1} and important effects are found at large total angular momentum. The solution introduced

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in ECIS [4] was to use DWBA results from a *nuclear matching point* to infinity, this matching point being chosen such that results does not depend upon it. Above some total angular momentum, J, the resolution of the radial equations is replaced by analytical integrals between zero and infinity of products of regular Coulomb functions with r^{-L-1} [5, 6]. The versions of this stage, as ECIS79 can handle all the problems previously solved by the method.

Trying to avoid energy dependence for fast proton scattering, some people began to use a Dirac potential [7] for elastic scattering. There is no reason that the Dirac potential should not be treated like Schrödinger potential to describe inelastic scattering. Usual methods to solve Coupled Channels equations are quite heavy; the ECIS method is quite similar to the Schrödinger case. Therefore, the use of Dirac potentials has been introduced in ECIS88 [8, 9], but limited to scattering of particles with spin $\frac{1}{2}$ and computation using the iteration method. The method used previously for heavy ions allows to obtain correct results, using non relativistic Coulomb functions. Such problems are not limited to charged particles: for neutrons, they appears when the interaction of the anomalous magnetic moment with the Coulomb field is taken into account to describe the polarisation in the elastic scattering. Meantime, at Argonne National Laboratory, P. Moldauer developed the code ANLECIS, starting from ECIS79, to study effects of direct interactions on compound nucleus results; this work was incorporated in ECIS88. Note that compound nucleus calculations need all the solutions and that there is no more an evident advantage of the iteration method if the spin orbit interaction is not deformed.

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Some other additions should be done for experiments with exotic beams. Such beams are usually produced by extraction at 0 degrees after a reaction, which produces tensor polarisations very large for exotic nuclei with high spins. There are tensor polarisation effects on the total scattering cross-sections and on the total cross-section, even if they are generally too small to be measured. Angular distributions can be obtained with ECIS since a long time, but there is no precise computation of total values (integrated over the angles).

2 Dispersion relations for the potential.

The code ECIS96 can deal with more than one nuclear potential. Using the notations of Mahaux and Sartor [11], each of these potentials includes :

- a real potential $V_{HF}(r)$, considered as independent on energy by the code, including a volume and a surface term, with Woods-Saxon form factors, usual or "symmetrised",

- an imaginary potential $W(r;E)$, dependent on energy, including a volume and a surface term like the real potential, but with a geometry which can be different, defined from $E = -\infty$ to $E = \infty$ and symmetric with respect to the Fermi energy E_F,

- the real potential $\Delta V(r;E)$, dispersive contribution related to the imaginary potential $W(r,E)$ by :

$$\Delta V(r,E) = (E - E_F)\pi^{-1} \int_{-\infty}^{\infty} \frac{W(r,E')}{(E' - E)(E' - E_F)} dE' \qquad (1)$$

  where E_F is the Fermi energy.

The volume imaginary potential at the energy E in the center of mass system is assumed to be :

$$W_V(r,E) = f_v(r) W_v \frac{(E - E_F)^{n_v}}{(E - E_F)^{n_v} + b_v^{n_v}} \qquad (2)$$

where $f_v(r)$ is the volume form factor,n_v is some even integer (usually 2) and b_v some constant. The surface imaginary potential is assumed to be similar, but multiplied by a damping factor :

$$W_S(r,E) = f_s(r) W_s \frac{(E - E_F)^{n_s} e^{-c|E-E_F| - r(E-E_F)}}{(E - E_F)^{n_s} + b_s^{n_s}} \qquad (3)$$

where $f_s(r)$ is the surface form factor, n_s and b_s are parameters similar to those of the volume imaginary potential, the parameter c introduces an exponential decrease of the surface potential and r is a non-locality range parameter, inverse of the parameter E_b defined in equations (3.17) and (3.18) of Ref. [11].

The code can handle Dirac potentials. In that case, the volume and surface potentials are replaced by the scalar and the vector potentials. There is no prohibition to use dispersion relations but scalar and vector potentials are treated as the volume part of the Schrödinger potential.

After each potential, must be given :

- n_v power for volume or scalar potential (if 0, dispersion relations are not used for this form factor),

- n_s power for surface or tensor potential (if 0, dispersion relations are not used for this form factor),

- E_0 reference energy in the center of mass system, that is the energy at which the depths have the values read with the geometry of the form factor (if 0, it is the energy of the first level using this potential),

- E_F Fermi energy, around which the energy dependence is symmetric (if 0, the value used is -6.8 MeV),

- b_v constant for volume or scalar potential,
- b_s constant for surface or tensor potential,
- c constant of the exponential decrease of a surface potential,
- r non-locality range parameter for which the default value 0.0125 is not in the code to allow to use 0.

For each level, the strengths of the real and the imaginary surface and volume potentials are computed and divided by the strength of the imaginary potential at the reference energy. One (1.) is subtracted to the imaginary strengths. These four numbers, which are printed, are the corrections due to dispersion relations to the form factors. There is the possibility to correct also the transition form-factors, using the mean value of the corrections of the two levels between which they act.

Up to now, it was possible to use a different potential for each level. However, for compound nucleus, many uncoupled states can be involved in the calculation. Up to `ECIS94` a potential could be used for each of them, but it is no more possible with `ECIS95` in which many levels are introduced by the code to take into account a continuum. It is chiefly to describe compound nucleus continuum that dispersion relations have been introduced in the code, because they should affect strongly results near threshold.

3 Evaluation of the integrals.

For a surface potential, using $U = E' - E_F$ and $F = E - E_F$, the integral given by Equ (1) and Equ (3) is for n even :

$$\frac{F}{\pi}\int_{-\infty}^{\infty}\frac{U^{n-1}e^{-c|U|-rU}}{(U^n+b^n)(U-F)}dU =$$

$$\frac{F}{\pi}\int_{0}^{\infty}\frac{U^{n-1}e^{-(c+r)U}}{(U^n+b^n)(U-F)}dU - \frac{F}{\pi}\int_{0}^{\infty}\frac{U^{n-1}e^{-(c-r)U}}{(U^n+b^n)(U+F)}dU \qquad (4)$$

and the same expression without exponential for a volume potential. First of all, the integrand must be replaced by its expression in terms of poles and residues :

$$\frac{F}{\pi}\quad\frac{U^{n-1}}{(U^n+b^n)(U-F)} = \frac{1}{\pi}\quad\sum_{j=1}^{j=n}\frac{r_j}{U-p_j} + \frac{1}{\pi}\quad\frac{F^n}{F^n+b^n}\quad\frac{1}{U-F} \qquad (5)$$

where the p_j are the n zeros of (U^n+b^n) and the r_j their residue, that is :

$$p_j = b\,e^{i\frac{(2j-1)}{n}\pi}, \qquad r_j = \frac{F}{n}\quad\frac{1}{p_j-F} \qquad (6)$$

For even values of n, these poles are all complex numbers, complex conjugate two by two. Besides these n poles, there is a real pole in each part of the

second member of Equ (4). The contribution of a pole is :

$$\int_0^\infty \frac{r_j}{U-p_j}\, e^{-(c\pm r)U} dU = r_j\, e^{-(c\pm r)p_j} \int_{-(c\pm r)p_j}^\infty \frac{e^{-z}}{z} dz = r_j\, e^{-(c\pm r)p_j} E_1(-(c\pm r)p_j) \tag{7}$$

where $E_1(z)$ is the *Exponential Integral Function*[12]. This function can be obtained using the series expansion given by Equ(5.1.11) of Ref. [12], which is :

$$E_1(z) = -\gamma - \ln z - \sum_{n=1}^{\infty} \frac{(-z)^n}{n!n} \tag{8}$$

where $\gamma = .57721566490153$ is the Euler number. This has to be done for the $n/2$ complex poles in the upper plane, duplicating the real part of the result. This formula is assumed to be always good because the variable z is equal to b multiplied by a small number. For the real pole situated at $\pm(E - E_F)$, the series expansion is used only for $-30 < z < 10$. Outside this domain (for large values of E), the continued fraction given by Equ(5.1.22) of Ref. [12], which is :

$$E_1(z) = e^{-z}\Big(\frac{1}{z+}\,\frac{1}{1+}\,\frac{1}{z+}\,\frac{2}{1+}\,\frac{2}{z+}\ldots\Big) \tag{9}$$

gives easily the result.

The complex poles are independent of E, but their contribution is multiplied by a function of E. So, the whole calculation is done at each energy.

For the volume form-factor, there is no exponential. The real pole does not contribute. A complex pole at p_j gives a contribution of $i\pi$ with the sign of imaginary part of p_j.

4 An example.

Let us consider the elastic scattering of 3.0 MeV neutrons on ^{184}W described in the rotational model with $R\beta_2 = 1.49781$, $R\beta_4 = -.4013$ and consider only a compound nucleus neutron continuum starting at zero energy. The real potential is described by a volume form factor with $V_v = 46.0$ MeV, $r_v = 1.26$ fermi and $a_v = .63$ fermi; there is a spin orbit potential with the same geometry and a strength $V_{LS} = 6.0$ MeV. The imaginary potential is a volume form factor with the same geometry and the strength $W_v = .33$ MeV and a surface form factor with $W_s = 6.0$ MeV, $r_v = 1.28$ fermi and $a_v = .47$ fermi. The level density in the continuum is described by a Gilbert and Cameron formula [13] with 20.69 as level density parameter for S-wave resonance spacing, 4.62 as matching energy for the two density formula shifted by pairing energy, .53 as nuclear temperature, 5.3 as spin cut–off parameter and $-.15$ as energy shift. For the dispersion relations, the Fermi energy is -6.8 MeV, the powers are $n_v = n_s = 2$, the constants are $b_v = 63.$ MeV and $b_s = 15.5$ MeV, the coefficient of exponential decrease of the surface potential is $c = .036$ and there is no non–locality range parameter.

On Figure 1a) are presented the coefficients of the real and the imaginary, volume and surface potentials. These coefficients multiply the form factors of

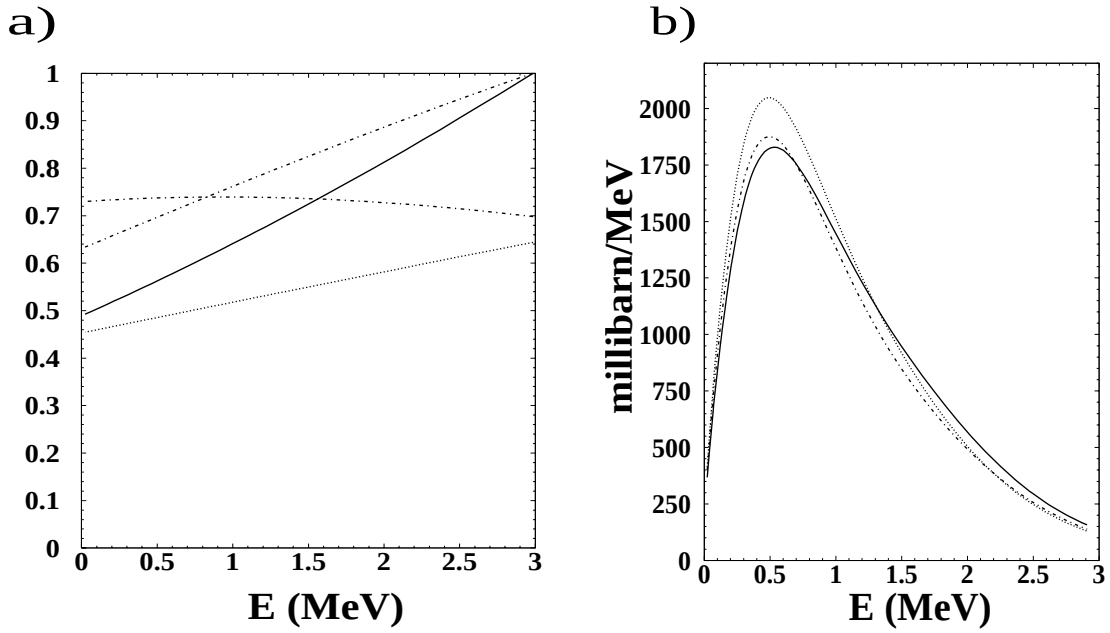

Figure 1: a) Coefficients of the correction to the potentials coming from dispersion : the curves are respectively the multiplicative factor of the volume imaginary potential (full line), the real volume potential divided by 10 (dotted line),the multiplicative factor of the surface imaginary potential (dashed line) and the real surface potential (mixed line). b) Comparison of results obtained with `ECIS96` (full line) and two options of `ECIS95` for a simple model calculation taking into account only elastic scattering, and this continuum of compound nucleus: use for the continuum of the potential of elastic scattering (dotted line) and of the maximum of these curves (mixed line).

the imaginary potential which include already the depths, .33 MeV and 6.0 MeV respectively. For convenience, the coefficient of the real volume potential has been divided by 10.; due to the smallness of the volume imaginary potential, it introduce corrections of the order of 2.0 MeV.

On Figure 1b) are presented results of the calculation as a function of the energy of the outgoing particles :

- the full curve is the result obtained with dispersion relations,
- the dotted curve is the result obtained with the potential used for the elastic scattering (including real dispersive potential) at all the energies,
- the dashed curve is the result obtained by using for the continuum the potential which was used at the maximum of the full curve, that is at $E = .555$ MeV.

5 Use of exotic beams.

Beams of excited particles are generally obtained by accelerating the product of a reaction outgoing at 0 degrees. Such a beam has strong tensor polarisations and there are consequences in the reactions studied with it [14].

If a beam of particles with spin 2^+ is produced by scattering α on a nucleus with spin 0^+, this beam is a pure states of particle with helicity 0 along the direction of the beam. Its polarisation is described by some parameters $t^{\lambda,\mu}$,

which are, in the more general case of a beam of integer spin S :

$$t^{\lambda,\mu} = (-)^S \sqrt{2S+1} < S\, S\, 0\, 0 | \lambda \mu > \qquad (10)$$

where $< \ldots | \ldots >$ is a Clebsch-Gordan coefficient which shows that the polarisation parameters are limited to those with $\mu = 0$ and λ even. For a beam of particles with spin 2+, these parameters are $t^{20} = -\sqrt{10/7} = -1.19522851$ and $t^{40} = \sqrt{18/7} = 1.60356734$. Limits of variations of this kind of parameter depend on the spin: for $S = 1$, $-\sqrt{2} < t^{20} < \sqrt{1/2}$ but for $S = 2$, $-\sqrt{10/7} < t^{20} < \sqrt{10/7}$ and $-\sqrt{8/7} < t^{40} < \sqrt{20/7}$.

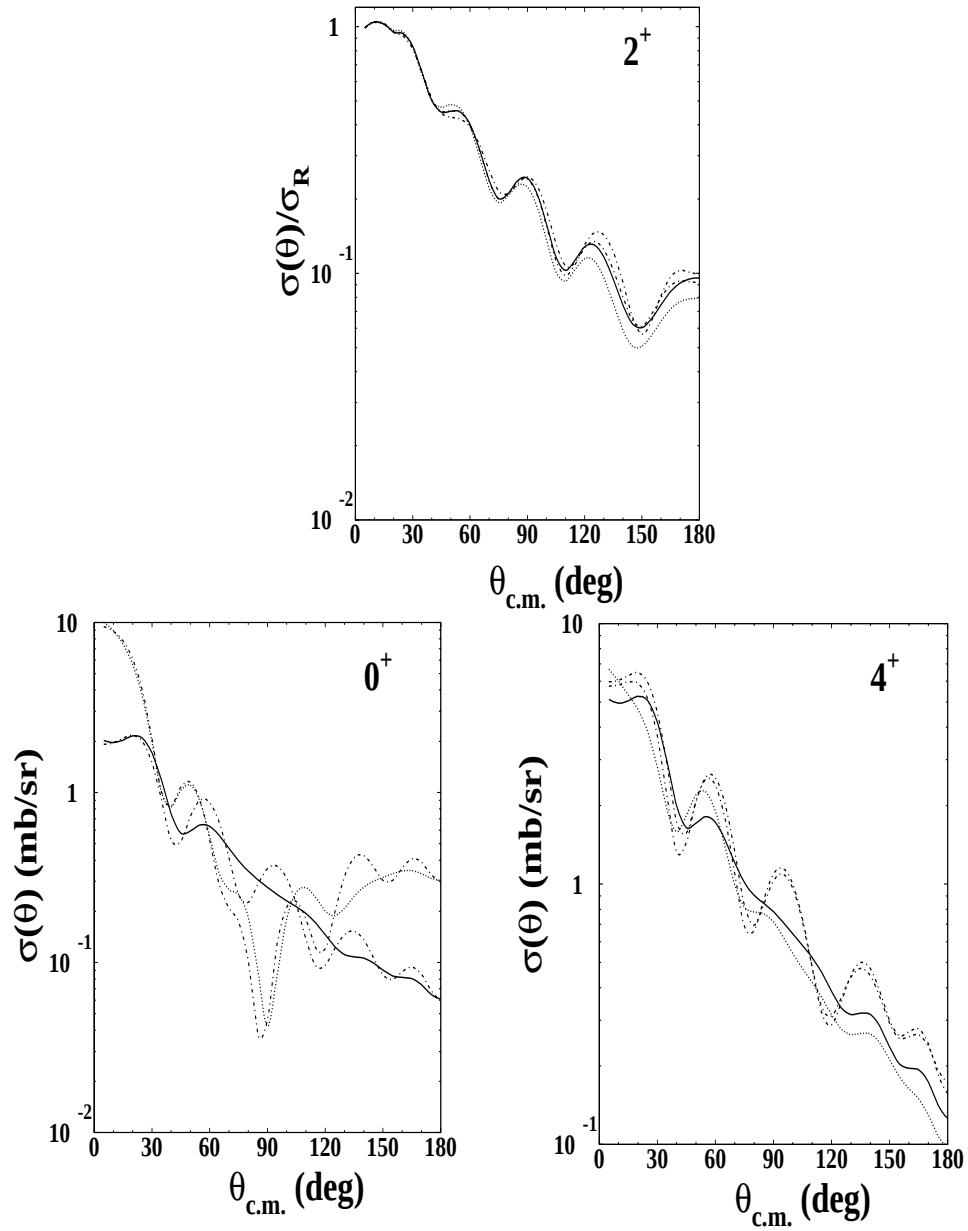

Figure 2: Elastic scattering an inelastic scattering of protons on a 2^+ : unpolarised cross section (full line) and cross section on a 2^+ obtained by spinless forward scattering (dotted line);the dashed line and the mixed line are respectively the same results with a change of sign of the deformations.

To show the consequences, let us consider the scattering of 20 MeV protons on the $0^+ - 2^+ - 4^+$ states of ^{184}W described by the rotational model with the same deformations as before. There is a volume central potential $V_v = 46.0$ MeV, $W_v = 8.0$ MeV with the geometry already used for neutrons and the same spin-orbit potential. The parameters t^{20} and t^{40} describing the polarisation of the 2^+ residual nucleus have the values given above at 0 degrees with a shift of less than .03 at 5 degrees. All the other parameters vanish when summed up on all directions around the incident beam.If protons are scattered on the 2^+ beam obtained in these conditions, the cross section is :

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{d\sigma}{d\Omega}(\theta)_{n.p.}[1 + t^{20}T^{20}(\theta) + t^{40}T^{40}(\theta)] \tag{11}$$

where $\frac{d\sigma}{d\Omega}(\theta)_{n.p.}$ is the cross-section for an unpolarised incident beam, $T^{20}(\theta)$ and $T^{40}(\theta)$ are the description of the polarisation of the outgoing beam in the inverse reaction. In particular, if the final state is the 0^+, $T^{20}(0) = t^{20}$ and $T^{40}(0) = t^{40}$ and the cross-section at 0 degrees (also at 180 degrees) is five times the unpolarised cross-section.

Figure 2 shows the cross-sections obtained in this $2^+ - 0^+ - 4^+$ calculation, the corresponding unpolarised cross sections and the same results obtained with a change of sign of all the deformations. It can be seen that effects are quite small on the elastic scattering, large for the 4^+ and tremendous for the 0^+.

If the exotic beam of spin S_f is produced by scattering of particle with spin s_i on a target of spin S_i with a final particle of spin s_f, the helicity of the beam is limited by $|m_{max}| \leq s_i + S_i + s_f$ and the non vanishing polarisation parameters $t^{\lambda 0}$ for even values of $0 < \lambda \leq 2S_f$ are restricted by the relations :

$$\sum_{\lambda=2}^{2S_f} < S_f\, S_f\, m\, -m|\lambda\, 0 > \, t^{\lambda 0} = -1 \tag{12}$$

for all values $m > m_{max}$. On another side, if the production of the beam is obtained by a transfer characterised by some ΔS, ΔL and ΔJ, a `DWBA` calculation gives $t^{\lambda 0}$ for $\lambda > 2\Delta L$ or $\lambda > 2\Delta J$. Coupled channel calculations give non zero but small values for these large λ's. With $S_i \neq 0$ and $s_i + s_f$ small, the description of the exotic beam can be complete.

Such cross sections are treated by `ECIS` as polarisations : the output prints only their ratio to the unpolarised cross-section and only this ratio can be given as experimental data. Their description can be given by the $t^{\lambda\mu}$ (taking into account $t^{00} = 1.$) or by the values of the elements of the density matrix. A precise value of the integrated effect of the $t^{\lambda\mu}$ an elastic, inelastic or the total reaction cross section is not computed : such effect exists for $\mu = 0$ and even values of λ.

6 Conclusions.

Since `ECIS94`, the main modification of the code is the introduction of the continuum for compound nucleus, which is quite independent of the original

purpose of the programme. The introduction of dispersion relations for the potential showed very important effects for this problem but needed a deep rewriting of subroutines. Some other changes have been done at the same time :

- for identical particle and target with non-zero spin, the symmetrised equations are solved instead of the symmetrisation of the results; the previous approach was not compatible with compound nucleus (but in which case should a compound nucleus be formed from an identical particle and target?),
- a maximum J-value has been introduced for compound nucleus calculation with a test which stops the calculation when contributions are smaller than ϵ^2 (direct interaction is stopped when contributions are less than ϵ),
- computation of compound nucleus angular distribution go back to the usual method via Legendre polynomials instead of the helicity formalism used before; the products of a 3-$j$coefficient and two 6-j coefficients needed in the most general case for the initial and for the final channel are computed by recurrence.

The topic of exotic beams shows that useful features were already in `ECIS75` but new features which are not in `ECIS96` can be needed.

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