

# Microscopic Optical Potentials for Elastic Nucleon-Nucleus Scattering

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The theoretical approach to the elastic scattering of a nucleon from a nucleus, pioneered by Watson [1], made familiar by Kerman, McManus, and Thaler (KMT) [2] and further developed as the spectator expansion [3, 4, 5, 6] is now being applied with striking success. In a similar vein, a slightly different approach to the multiple scattering expansion within the KMT framework is being pursued by the Surrey group [7]. The theoretical motivation for the spectator expansion derives from our present inability to calculate the full many-body problem. In this case an expansion is constructed within a multiple scattering theory predicated upon the idea that two-body interactions between the projectile and the target nucleons inside the nucleus play the dominant role. In the spectator expansion the first order term involves two-body interactions between the projectile and one of the target nucleons, the second order term involves the projectile interacting with two target nucleons and so forth. Hence the expansion derives the ordering from the number of target nucleons interacting directly with the projectile, while the residual target nucleus remains ‘passive’. Due to the many-body nature of the free propagator for the the projectile + target system it is necessary to detail certain choices made with respect to the ordering in the spectator series. In our approach a theoretical treatment of this many-body propagator as affected by the residual target nucleus is included in first order.

The calculation of the optical potential relies on two basic input quantities. One is the fully-off-shell NN t-matrix, which represents the current understanding of the nuclear force, and the other is the nuclear wave functions of the target, representing the best understanding of the ground state of the target nucleus. To account for the modifications of the free propagator inside the nucleus, the same mean field potentials are used from which the ground state wave function of the target are derived. There are **no** adjustable free parameters present in this calculation.

The standard approach to elastic scattering of a strongly interacting projectile from a target of  $A$  particles is the separation of the Lippmann-Schwinger equation for the transition amplitude

$$T = V + VG_0(E)T \tag{1}$$

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into two parts, namely an integral equation for  $T$ :

$$T = U + UG_0(E)PT, \quad (2)$$

where  $U$  is the optical potential operator and defined by a second integral equation

$$U = V + VG_0(E)QU. \quad (3)$$

In the above equations the operator  $V = \sum_{i=1}^A v_{0i}$  consists of the two-body NN potential  $v_{0i}$  acting between the projectile and the  $i$ th target nucleon. The free propagator  $G_0(E)$  for the projectile+target system is given by  $G_0(E) = (E - H_0 + i\epsilon)^{-1}$ , and the Hamiltonian for the  $(A+1)$  particle system by  $H = H_0 + V$ . Here the free Hamiltonian is given by  $H_0 = h_0 + H_A$ , where  $h_0$  is the kinetic energy operator for the projectile and  $H_A$  stands for the target Hamiltonian. Defining  $|\Phi_A\rangle$  as the ground state of the target, we have  $H_A|\Phi_A\rangle = E_A|\Phi_A\rangle$ .

The operators  $P$  and  $Q$  in Eqs. (1) and (2) are projection operators with  $P + Q = 1$ , and  $P$  being defined such that Eq. (2) becomes a one-body equation. In this case,  $P$  is conventionally taken to project on the elastic channel, such that  $[G_0, P] = 0$ , and is defined as  $P = |\Phi_A\rangle\langle\Phi_A|/\langle\Phi_A|\Phi_A\rangle$ . With these definitions, the transition operator for elastic scattering can be defined as  $T_{el} = PTP$ , in which case Eq. (2) can be written as

$$T_{el} = PUP + PUPG_0(E)T_{el}. \quad (4)$$

The fundamental idea of the spectator expansion for the optical potential is an ordering of the scattering process according to the number of active target nucleons interacting directly with the projectile. The first order term involves two-body interactions between the projectile and one of the target nucleons, *i.e.*  $U = \sum_{i=1}^A \tau_i$ , where the operator  $\tau_i$  is derived to be

$$\begin{aligned} \tau_i &= v_{0i} + v_{0i}G_0(E)Q\tau_i \\ &= v_{0i} + v_{0i}G_0(E)\tau_i - v_{0i}G_0(E)P\tau_i \\ &= \hat{\tau}_i - \hat{\tau}_iG_0(E)P\tau_i. \end{aligned} \quad (5)$$

Here  $\hat{\tau}_i$  is defined as the solution of

$$\hat{\tau}_i = v_{0i} + v_{0i}G_0(E)\hat{\tau}_i. \quad (6)$$

It should be noted that the above equations all follow in a straightforward derivation and correspond to the first order Watson scattering expansion [1].

For elastic scattering only  $P\tau_iP$  (from Eq. (5)) or equivalently

$$\langle\Phi_A|\tau_i|\Phi_A\rangle = \langle\Phi_A|\hat{\tau}_i|\Phi_A\rangle - \langle\Phi_A|\hat{\tau}_i|\Phi_A\rangle \frac{1}{(E - E_A) - h_0 + i\epsilon} \langle\Phi_A|\tau_i|\Phi_A\rangle, \quad (7)$$

needs to be considered. Since Eq. (7) is a one-body integral equation, the principal problem is to find a solution of Eq. (6), which still has a many-body character due to  $G_0(E) = (E - h_0 - H_A + i\epsilon)^{-1}$ . If the propagator  $G_0(E)$  is expanded in the spirit of the spectator expansion within a single particle description, one obtains in first order [6, 9]

$$G_i(E) = [(E - E^i) - h_0 - h_i - W_i + i\epsilon]^{-1}. \quad (8)$$

Here  $h_i$  is the kinetic energy of the  $i$ th target particle and  $W_i = \sum_{j \neq i} v_{ij}$  represents the force acting between the struck target nucleon and the residual (A-1) nucleus. Then the operator  $\hat{\tau}_i$  of Eq.(6) can be written as

$$\begin{aligned} \hat{\tau}_i &= v_{0i} + v_{0i}G_i(E)\hat{\tau}_i \\ &= t_{0i} + t_{0i}g_iW_iG_i(E)\hat{\tau}_i. \end{aligned} \quad (9)$$

Here the operators  $t_{0i}$  and  $g_i$  are defined as  $t_{0i} = v_{0i} + v_{0i}g_it_{0i}$  and  $g_i = [(E - E^i) - h_0 - h_i + i\epsilon]^{-1}$ . The second term of Eq. (9), namely  $t_{0i}g_iW_iG_i(E)\hat{\tau}_i$ , can be considered as modification of the free scattering process given by  $t_{0i}$  due to the presence of the nuclear medium in the first order spectator expansion. The structure of Eq.(9) also shows that already the first order term of the optical potential exhibits a three-body character [6, 8]. Our numerical treatment of this term includes the full spin-structure of this last term in Eq. (9), but not the full three-body kinematics and is given in detail in Ref. [9].

In lowest order, the operator  $\hat{\tau}_i$  of Eq. (9) is given by  $\hat{\tau}_i \approx t_{0i}$ , which corresponds to the conventional impulse approximation. the matrix element  $\langle \Phi_A | \tau_i | \Phi_A \rangle$  given in Eq. (7) represent the full-folding optical potential and is given explicitly as

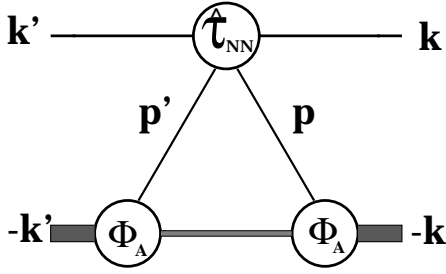
$$\langle \mathbf{k}' | U | \mathbf{k} \rangle = \langle \mathbf{k}' \Phi_A | \sum_{\alpha=p,n} \tau_\alpha | \mathbf{k} \Phi_A \rangle, \quad (10)$$

where  $\alpha$  represents the sum over the target protons and neutrons. Since  $\langle \mathbf{k}' | U | \mathbf{k} \rangle$  is the solution of the sum of the one-body integral equations represented by Eq. (7), it is sufficient to consider the driving term

$$\langle \mathbf{k}' | \hat{U} | \mathbf{k} \rangle = \langle \mathbf{k}' \Phi_A | \sum_{\alpha=p,n} \hat{\tau}_\alpha | \mathbf{k} \Phi_A \rangle, \quad (11)$$

where  $\hat{\tau}_\alpha$  is given by Eq. (9). Inserting a complete set of momenta for the struck target nucleon before and after the collision we obtain

$$\hat{U}(\mathbf{k}, \mathbf{k}') = \sum_{\alpha=p,n} \int d^3\mathbf{p}' d^3\mathbf{p} \langle \mathbf{k}' \mathbf{p}' | \hat{\tau}_\alpha(\epsilon) | \mathbf{k} \mathbf{p} \rangle \rho_\alpha \left( \mathbf{p}' + \frac{\mathbf{k}'}{A}, \mathbf{p} + \frac{\mathbf{k}}{A} \right) \delta^3(\mathbf{k}' + \mathbf{p}' - \mathbf{k} - \mathbf{p}). \quad (12)$$

**Fig. 1****FIG. 1:**Diagram for the optical potential matrix element for the single scattering term

$$\hat{U}(\mathbf{q}, \mathbf{K}) = \sum_{\alpha=p,n} \int d^3\mathbf{P} \left\langle \mathbf{k}', \mathbf{P} - \frac{\mathbf{q}}{2} - \frac{\mathbf{K}}{A} \mid \hat{\tau}_{\alpha}(\epsilon) \mid \mathbf{k}, \mathbf{P} + \frac{\mathbf{q}}{2} - \frac{\mathbf{K}}{A} \right\rangle \rho_{\alpha} \left( \mathbf{P} - \frac{A-1}{A} \frac{\mathbf{q}}{2}, \mathbf{P} + \frac{A-1}{A} \frac{\mathbf{q}}{2} \right). \quad (13)$$

The NN amplitude  $\hat{\tau}_{\alpha}$  in Eq. (13) is evaluated in the zero momentum frame of the nucleon-nucleus system. In order to relate this amplitude to the corresponding matrix element evaluated in the zero momentum frame of the two nucleons, we have to introduce a Møller factor for the frame transformation [11] and obtain as final expression for the full-folding optical potential [12].

$$\hat{U}(\mathbf{q}, \mathbf{K}) = \sum_{\alpha=p,n} \int d^3\mathbf{P} \eta(\mathbf{P}, \mathbf{q}, \mathbf{K}) \hat{\tau}_{\alpha} \left( \mathbf{q}, \frac{1}{2} \left( \frac{A+1}{A} \mathbf{K} - \mathbf{P} \right); \epsilon \right) \rho_{\alpha} \left( \mathbf{P} - \frac{A-1}{A} \frac{\mathbf{q}}{2}, \mathbf{P} + \frac{A-1}{A} \frac{\mathbf{q}}{2} \right). \quad (14)$$

Here the arguments of  $\hat{\tau}_{\alpha}$  are  $\mathbf{q} = \mathbf{k}' - \mathbf{k} = \mathbf{K}' - \mathbf{K}$  and  $\frac{1}{2}(\mathbf{K}' + \mathbf{K}) = \frac{1}{2}(\frac{A+1}{A}\mathbf{K} - \mathbf{P})$  and

$$\eta(\mathbf{P}, \mathbf{q}, \mathbf{K}) = \left[ \frac{E_N(\mathbf{K}') E_N(-\mathbf{K}') E_N(\mathbf{K}) E_N(-\mathbf{K})}{E_N(k') E_N(\mathbf{P} - \frac{\mathbf{q}}{2} - \frac{\mathbf{K}}{A}) E_N(k) E_N(\mathbf{P} + \frac{\mathbf{q}}{2} - \frac{\mathbf{K}}{A})} \right]. \quad (15)$$

The free two-nucleon amplitude  $\hat{\tau}_{\alpha}$  is calculated from the free NN t-matrix according Eq. (9) at an appropriate energy  $\epsilon$ . In principle, this energy should be the beam energy minus the kinetic energy of the center of mass of the interacting pair less the binding energy of the struck particle. Following this argument,  $\epsilon$  should be coupled to the integration variable  $\mathbf{P}$ . The full-folding calculations of Refs. [13, 14] are carried out along this vain. For our calculations we take a different approach, namely we fix  $\epsilon$  at the two-body center-of-mass (c.m.) energy corresponding to free NN scattering at the beam energy so that the same laboratory energy applies to the two-body and nuclear scattering. This approach has been applied in earlier work [15] and is also used in the work of the Surrey Group [7].

The evaluation of the full-folding optical potential as given in Eq. (14) requires a nuclear density matrix, which in a single particle picture is given as

$$\rho_\alpha(\tilde{\mathbf{p}}', \tilde{\mathbf{p}}) = \sum_i \Psi_{\alpha,i}^\dagger(\tilde{\mathbf{p}}') \Psi_{\alpha,i}(\tilde{\mathbf{p}}), \quad (16)$$

where  $\Psi_{\alpha,i}(\tilde{\mathbf{p}})$  are the wave functions describing the single particle nuclear ground state. The index  $\alpha$  stands for protons and neutrons, respectively, and the total nuclear ground state is given by the sum of the two. In order to achieve consistency with our formulation of incorporating effects of the ‘nuclear medium’ on the scattering process we choose as model density matrices the ones derived from the same nuclear mean fields  $W_i$  as given in Eq. (9). The models used are a non-relativistic reduction of a Dirac-Hartree model [16] and a non-relativistic Hartree-Fock-Bogolyubov (HFB) structure calculation [17, 18]. The Dirac-Hartree calculation is a spherical solution of the one-body Dirac equation assuming a scalar potential and the time component of a vector potential field. The nonrelativistic HFB microscopic nuclear structure calculation uses the parameterized effective finite-range, density dependent Gogny D1S effective NN interaction. The parameter of the Gogny D1S interaction are fitted to a certain set of stable nuclei.

The density matrix derived from the Dirac Hartree calculation is given by

$$\rho_\alpha(\mathbf{r}', \mathbf{r}) = \sum_{n\nu} \left[ \frac{G_{\alpha,t_z,n,\nu}(r')}{r'} \frac{G_{\alpha,t_z,n,\nu}(r)}{r} + \frac{F_{\alpha,t_z,n,\nu}(r')}{r'} \frac{F_{\alpha,t_z,n,\nu}(r)}{r} \right] \frac{2j+1}{2l+1} \sum_{m_l} Y_l^{*m_l}(r') Y_l^{m_l}(r), \quad (17)$$

where  $n$  stands for the principal quantum number,  $\nu$  for the angular momentum quantum numbers  $J$  and  $l$  and  $t_z$  for the z-component of the isospin. After a double Fourier transform we obtain

$$\begin{aligned} \rho_\alpha(\tilde{\mathbf{p}}', \tilde{\mathbf{p}}) &= \frac{1}{2\pi^2} \sum_J (2J+1) \sum_l P_l(\cos \theta_{\tilde{\mathbf{p}}, \tilde{\mathbf{p}}'}) \\ &\left[ \int dr' r' j_l(\tilde{p}' r') F_{\alpha,t_z,n,\nu}(r') \int dr r j_l(\tilde{p} r) F_{\alpha,t_z,n,\nu}(r) + \right. \\ &\left. \int dr' r' j_l(\tilde{p}' r') G_{\alpha,t_z,n,\nu}(r') \int dr r j_l(\tilde{p} r) G_{\alpha,t_z,n,\nu}(r) \right]. \end{aligned} \quad (18)$$

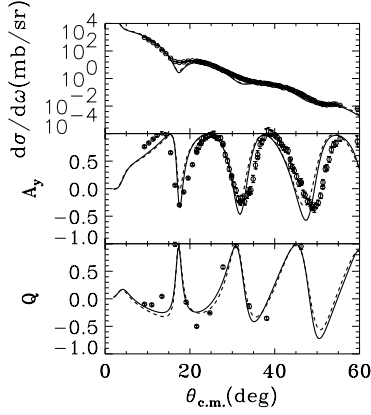
Details of the derivation can be found in Ref. [12]. The density matrix  $\rho_\alpha(\tilde{\mathbf{p}}', \tilde{\mathbf{p}})$  given in Eq. (18) is defined in the rest frame of the nucleus. For our calculation of the full-folding optical potential, we need to evaluate the function at the corresponding momenta in the nucleon-nucleus frame. This is facilitated by variable transformations  $\mathbf{p} = \tilde{\mathbf{p}} - \frac{\mathbf{k}}{A}$  and  $\mathbf{p}' = \tilde{\mathbf{p}}' - \frac{\mathbf{k}'}{A}$ , which takes into account recoil. For the calculation of the density matrices derived from a non-relativistic Hartree-Fock Bogolyubov (HFB) calculation based on the Gogny-D1S NN interaction we employ essentially the same procedure as described above. The wave functions are created in r-space by a code provided by Berger [17] and are represented in a axially symmetric harmonic oscillator basis. A double Fourier transform is then performed using the oscillator basis and summing over all harmonic oscillator quantum numbers.

The study of elastic scattering of neutrons and protons from spin-zero target nuclei presented here are strictly first order in the spectator expansion. The full-folding optical potential is calculated as given in Eq. (15) using either the DH or HFB model densities. The calculations for scattering at energies smaller than 200 MeV take into account the coupling of the struck target nucleon to the residual nucleus via the mean field potential  $W_i$ , which is chosen to be consistent with the model density employed. Details of this procedure are given in Refs. [6, 9]. In all cases (unless specified otherwise the calculations presented here use the free NN t-matrix based upon the full Bonn potential [19]. It is also to be understood that we perform all spin summations in obtaining  $\hat{U}(\mathbf{q}, \mathbf{K})$ . This reduces the required NN t-matrix elements to a spin independent component (corresponding to the Wolfenstein amplitude A) and a spin-orbit component (corresponding to the Wolfenstein amplitude C). Since we are assuming that we have spin saturated nuclei, the components of the NN t-matrix depending on the spin of the struck nucleon vanish.

In order to assess the importance of the off-shell character of the nuclear density matrix, we compare the full-folding calculations with the commonly used optimum factorized or off-shell ‘ $t\rho$ ’ approximation [10, 20]. The latter is obtained by observing that the nuclear size is significantly larger than the range of the NN interaction and thus to amplitude  $\hat{\tau}_\alpha$  is expected to be the slower varying quantity in the integral of Eq. (15). One then expands  $\hat{\tau}_\alpha$  (including the factor  $\eta(\mathbf{q}, \mathbf{K}, \mathbf{P})$ ) in  $\mathbf{P}$  about a fixed value  $\mathbf{P}_0$ , which is determined by requiring that the contribution of the first derivative term is minimized. After the integration over the density matrix, producing the diagonal density profile  $\rho_\alpha(q)$ , the ‘optimum factorized’ or ‘off-shell  $t\rho$ ’ form of the optical potential is given by

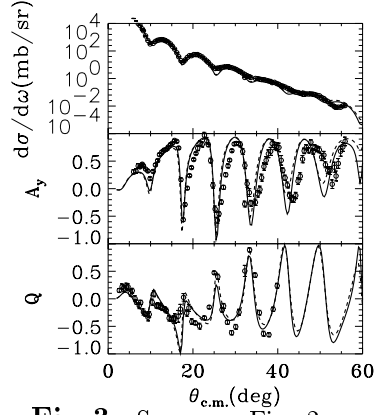
$$\hat{U}_{fac}(\mathbf{q}, \mathbf{K}) = \sum_{\alpha=p,n} \eta(\mathbf{q}, \mathbf{K}) \hat{\tau}_\alpha \left( \mathbf{q}, \frac{A+1}{2A} \mathbf{K}, \epsilon \right) \rho_\alpha(q). \quad (19)$$

Here the non-local character of the optical potential is solely determined by the off-shell NN t-matrix. In Figs. 2 and 3 we show the elastic scattering observables for proton scattering from  $^{40}\text{Ca}$  and  $^{208}\text{Pb}$  based on full-folding optical potentials and compare the to the corresponding calculations using the optimum factorized form. The figures show that the optimum factorized form is quite a good representation of the full-folding integral, even at lower energies. In the case of proton scattering from  $^{208}\text{Pb}$  at 65 MeV, the inclusion of the off-shell structure of the nuclear density matrix makes the nucleus appear slightly larger, which becomes apparent in the shifted diffraction pattern of  $\frac{d\sigma}{d\Omega}$ . In Fig. 4 total neutron cross section data for  $^{16}\text{O}$ ,  $^{40}\text{Ca}$ ,  $^{90}\text{Zr}$ , and  $^{208}\text{Pb}$  are shown along with various calculations of  $\sigma_{tot}(E)$  at a number of energies. The open circles represent the full-folding calculations, which are all based on the DH model for the nuclear densities.

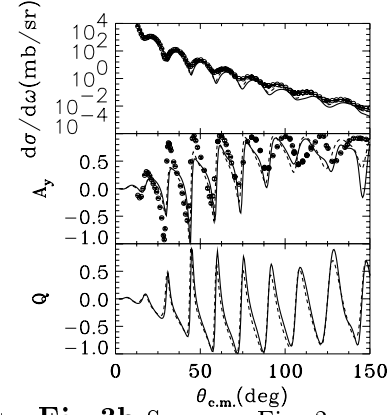


**Fig.2:**The angular distribution of the differential cross-section ( $\frac{d\sigma}{d\Omega}$ ), analyzing power ( $A_y$ ) and spin rotation function ( $Q$ ) are shown for elastic proton scattering from  $^{40}\text{Ca}$  at 200 MeV laboratory energy. The solid line represent the calculation performed with a first-order full-folding optical potential based on the DH density [16] and the full Bonn model [19], the dashed curve is based on the factorized, off-shell ‘ $t\rho$ ’ approximation. The data are taken from Ref. [24].

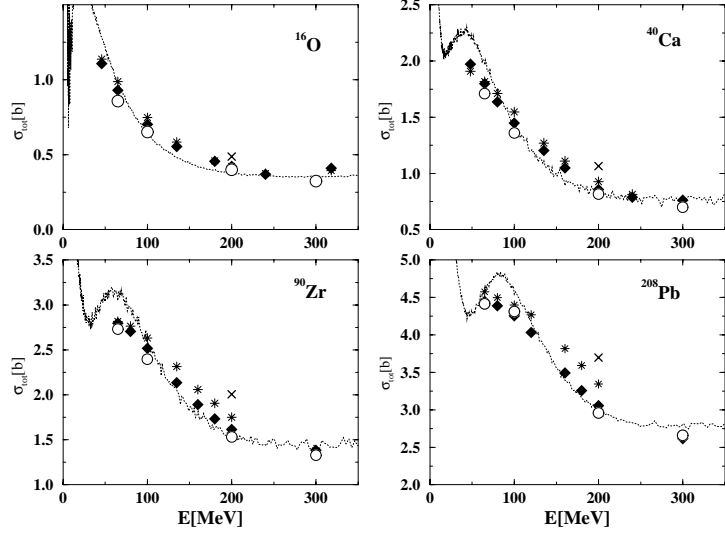
For energies below  $\leq 200$  MeV the modification of the free propagator through the DH mean field is included as described in Ref. [9]. It has been shown in Ref. [6] that for higher energies this modification of the free propagator becomes negligible. The solid diamonds represent calculations based on the factorized, off-shell ‘ $t\rho$ ’ form.



**Fig.3a:**Same as Fig. 2, except for  $^{208}\text{Pb}$  at 200 MeV. The data are from Ref. [25]

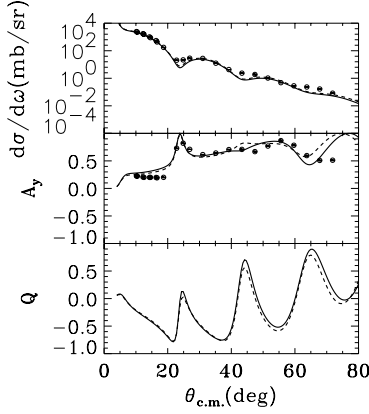


**Fig.3b:**Same as Fig. 2, except for  $^{208}\text{Pb}$  at 65 MeV. The data are from Ref. [26]

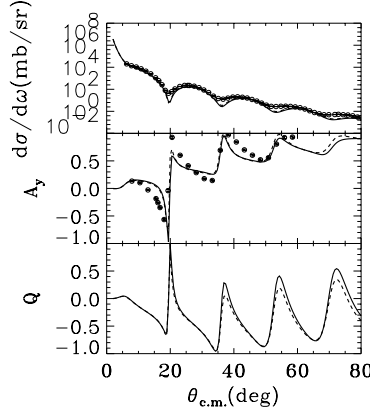


**Fig.4:**The neutron-nucleus total cross-sections for scattering from  $^{16}\text{O}$ ,  $^{40}\text{Ca}$ ,  $^{90}\text{Zr}$ , and  $^{208}\text{Pb}$  are shown as a function of the incident neutron kinetic energy. The dotted line represents the data taken from Ref. [27, 28]. The open circles correspond to the full-folding calculations using the full Bonn NN  $t$ -matrix [19] and the DH model [16] for the density. For energies below 200 MeV the coupling of the struck target nucleon to the residual nucleus via the mean field force  $W_i$  is taken into account. The solid diamonds represent the corresponding calculations using the optimum factorized, off-shell ‘ $t\rho$ ’ form. The stars represent calculations where the effects due to the nuclear medium are omitted. The crosses stand for calculations using a local, on-shell ‘ $t\rho$ ’ optical potential.

A general trend to be observed in Fig. 4 is the slightly lower value of  $\sigma_{tot}(E)$  obtained from the full-folding calculations compared to the factorized approximation. This trend is almost independent of the energy and the nucleus under consideration. The importance of including the coupling of the struck target nucleon to the residual nucleus becomes apparent at energies below 200 MeV. Calculations omitting this effect of the ‘nuclear medium’ on the free propagator are also given in Fig. 4 and indicated by stars. In order to point out the importance of including the off-shell structure of the NN t-matrix in the calculation of the optical potential, we also show in Fig. 4 the values of  $\sigma_{tot}(E)$  (indicated by crosses) obtained from a completely local potential. A local optical potential is obtained by imposing the on-shell constraint  $\mathbf{q} \cdot \mathbf{K} = 0$  on the NN t-matrix in Eq. (19), which leads then to the familiar form of a local optical potential,  $\hat{U}_{loc} = \hat{\tau}_\alpha(q)\rho_\alpha(q)$ .



**Fig.5:** The angular distribution of the differential cross-section ( $\frac{d\sigma}{d\Omega}$ ), analyzing power ( $A_y$ ) and spin rotation function ( $Q$ ) are shown for elastic proton scattering from  $^{40}\text{Ca}$  at 100 MeV laboratory energy[29] and  $^{90}\text{Zr}$  at 80 MeV laboratory energy[30]. In both cases the full-folding optical potentials are based on the full Bonn model [19] and the DH density (solid), as well as the HFB model (dashed) for the nuclear density. All calculations include the modification of the free propagator through the corresponding mean field.



**Fig.6:** The angular distribution of the differential cross-section ( $\frac{d\sigma}{d\Omega}$ ), analyzing power ( $A_y$ ) and spin rotation function ( $Q$ ) are shown for elastic proton scattering from  $^{40}\text{Ca}$  at 200 MeV laboratory energy. The calculations are based on the full-folding calculation and employ the NN t-matrices from the following potentials: Nijmegen I(solid), Nijmegen II (dashed), AV18 (dashed-dot) and CD Bonn (dotted).

In order to study the influence of the nuclear structure on the elastic observables, we show in Fig. 5 scattering of protons from  $^{40}\text{Ca}$  at 100 MeV and from  $^{90}\text{Zr}$  at 80 MeV. This choice of energies is mainly motivated by the availability of experimental data. In both cases the coupling of the struck target nucleon to the residual nucleus via the corresponding mean field force is taken into account. For the lighter nucleus  $^{40}\text{Ca}$  the elastic observables exhibit more sensitivity to the nuclear structure model than for the heavy nucleus  $^{90}\text{Zr}$ . A possible explanation for this behavior is the similarity of the two



models under consideration, namely the HFB and the DH model. Though both models have very different physical ingredients, we found that the resulting density matrices as well as the corresponding mean fields exhibit a very similar structure.

As already pointed out when discussing the total cross section for neutron scattering, the major part of the nonlocality in the nucleon-nucleus optical potential is due to the nonlocality in the NN t-matrix. In order to get a quantitative description of the experiment, the inclusion of the nonlocality of the NN t-matrix is absolutely necessary. Following along this vein, the next question to ask is whether p-nucleus scattering observables are sensitive to different off-shell structures in the NN t-matrices. To study this question, we show in Fig. 6 the scattering of protons from  $^{16}\text{Ca}$  at 200 MeV for four different NN potentials. These potentials, namely the Nijmegen models I and II [21], the Argonne potential AV18 [22] and the charge-dependent Bonn potential [23], describe the NN observables with  $\chi^2 \approx 1$ , which means that these potentials are essentially on-shell equivalent. The curves given in Fig. 6 show that the calculation of elastic p- $^{16}\text{O}$  observables exhibit no difference between these potentials. This implies that the optical potential for nucleon-nucleus scattering may not sample the off-shell NN t-matrix in regions far enough off the energy shell, where differences in the NN t-matrices occur.

In summary, we performed calculations of the first order optical potential for nucleon-nucleus scattering consistent within the first order spectator expansion of multiple scattering theory. Within this first order term, the effect of the nuclear medium is taken into account by including the coupling of the struck target nucleon to the residual nucleus via the same mean field force from which the nuclear density matrix is derived. One of the most noteworthy results is the correct prediction of the spin observables for elastic proton scattering from light ( $^{16}\text{O}$ ) as well as heavy ( $^{90}\text{Zr}$ ,  $^{208}\text{Pb}$ ) nuclei as low as 65 MeV. Another important result is the correct prediction of total cross section measurements for neutron scattering from light ( $^{12}\text{C}$ ,  $^{16}\text{O}$ ) as well as heavy ( $^{90}\text{Zr}$ ,  $^{208}\text{Pb}$ ) nuclei in the energy regime between 75 and 600 MeV. In the calculation of the full-folding optical potential the off-shell structure of the NN t-matrix enters as crucial ingredient. However, realistic, on-shell equivalent NN potentials have a very similar off-shell structure close to the on-shell condition, so that off-shell differences cannot be seen in the elastic observables for nucleon-nucleus scattering.

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