Neutron-Incident Phenomenological Dirac Optical Model Potential

Kenji Ishibashi, Shin-ichi Maruyama and Nobuhiro Shigyo
Department of Nuclear Engineering, Kyushu University
Hakozaki, Fukuoka, 812-81 Japan

Abstract

Some proton-incident data have been published on elastic-scattering and total-reaction cross sections in the intermediate energy region. Such results as well as experimental data obtained by polarized proton beams are useful for parametrizing global optical model potentials in detail. In contrast, neutron-incident experiments on elastic scattering are more difficult than proton ones, so that there is a limitation on neutron-incident data. It is, therefore, interesting to convert the proton-incident global optical model potentials into neutron-incident ones. The authors introduce *(N-Z)/A* dependent symmetry potential terms to the proton-incident optical potentials, and then obtain neutron-incident ones. The neutron potentials reproduce total cross sections in an acceptable degree. However, a comparison with potentials proposed by other authors brings about a confused situation in the sign of the symmetry terms.

1. Introduction

Studies have recently been made on the application of accelerators to medical facilities, spallation neutron sources and facilities of radioactive nuclear waste transmutation. Such studies require evaluated nuclear data libraries in the intermediate energy region. The nuclear data evaluation needs nuclear model calculations on the basis of the optical model potentials. For neutron-incident experiments on total reaction cross section, a number of data were obtained in the intermediate energy region. For elastic scattering, however, there are quite a few experimental data, and no data were taken by the use of polarized neutron beams. Shen et al.[1] determined the parameters of neutron-incident phenomenological Dirac optical model potentials; their validity is in question due to the situation that only limited quantities of the neutron data are available.

Unlike neutron-incident experiments, there are considerably more experimental data for proton incidence in the intermediate energy range. In particular, experimental data obtained by polarized proton beams are available on analyzing power and spin rotation function, so that potential parameters considering these additional data are more reliable. Phenomenological Dirac optical model potential parameters were obtained in a global form by Cooper et al.[2] Hence, it is of interest to convert the proton-incident potential parameters into neutron ones.

For construction of optical model potentials, there is an approach to determine the symmetry potentail term that includes *(N-Z)/A* dependence, where *N* is the number of neutrons, *Z* the atomic number and *A* the mass number. Kozack and Madland [3] made potential parametrization taking the symmetry term into account; they dealed with a target nucleus of ^{208}Pb for both proton and neutron incidence in the energy range of 95 to 300 MeV. In the present paper, an attempt is carried out to find how the symmetry term approach is useful for deriving global neutron-incident potentials from those of proton incidence. Potentials of interest in this study are in the range of neutron energies of 100 to 400 MeV, and targets of C to U.

2. Potentials based on the Dirac-Schrödinger equation

Because of treatment in the intermediate range, the Dirac equation is suitable for potential parametrization. The time-like four-vector potential U_V and the Lorentz scalar potential U_S are chosen in the following form:

$$U_V(r,E) = V_V(E) f_V(r,E) + i\left[W_V(E) g_V(r,E) + W_{VSP}(E) h_V(r,E)\right],$$

$$U_S(r,E) = V_S(E) f_S(r,E) + i\left[W_S(E) g_S(r,E) + W_{SSP}(E) h_S(r,E)\right],$$

In these potentials, $V_V(E), V_S(E), W_V(E)$ and $W_S(E)$ show the volume term, and are approximately proportional to the nucleon density. $W_{VSP}(E)$ and $W_{SSP}(E)$ indicate the surface term; they increase at the nuclear surface and are influential for low energy incidence. $f(r,E)$ and $g(r,E)$ stand for the nuclear density distribution; either Wood-Saxon[1] or symmetrized forms[2,3] are utilized. $h(r,E)$ is the differentiated function of $f(r,E)$, and accordingly takes a large value at the nuclear surface.

To simplify the calculation, the Dirac equation is often rearranged into a Schrödinger-type equation (Dirac-Schrödinger equation). Correspondingly, U_V and U_S are converted into the central potential U_{cent}, and the spin-orbit potential U_{so} as

$$\left[p^2 + 2E\left(U_{cent} + U_{SO}\bar{\cdot}\,\bar{L}\right)\right](r) = \left[\left(E - V_c\right)^2 - m\right](r),$$

$$U_{cent} = \frac{1}{2E}\left(2EU_V + 2mU_S - U_V^2 + U_S^2 - 2V_cU_V + 2EU_{Darwin}\right),$$

$$U_{so} = -\frac{1}{2EAr}\frac{\partial A}{\partial r},\ U_{Darwin} = -\frac{1}{2}\frac{1}{Ar^2}\left(\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}A\right) + \frac{3}{4}\frac{1}{A}\left(\frac{\partial}{\partial r}A\right),$$

$$A = \frac{E + m + U_S - U_V - V_c}{E + m}.$$

Partial wave analysis is made by the Dirac-Schrödinger equation, and cross sections such as elastic scattering and reaction are obtained.

3. Determination of potential parameters

3.1 Published global parametrizations

Cooper et al.[2] parametrized the global potentials for proton incidence. They used experimental data on elastic-scattering, analyzing power, spin rotation function and reaction cross sections for targets of ^{12}C to ^{208}Pb at incident proton energies of 20 to 1040 MeV. The potentials are expressed by quantities of the target mass number *A* and the incident kinetic energy and no explicit dependence was considered on the atomic number *Z*. The global potential reproduces the experimental data in general, but satisfactory results are not obtained for targets with mass numbers below 40 and those with highest mass such as Pb.

By the use of neutron data alone, Shen et al.[1] evaluated global potential parameters for nuclei of ^{12}C to ^{238}U in the energy range of 20 to 1000 MeV. At energies above 160 MeV, the potentials were determined only by the total cross sections due to lack of neutron-incident elastic scattering data. In the energy range of 200 to 500 MeV, the calculated total cross sections appreciably deviate from experimental data. In the nature of parameterization method, there may be a problem in reproduction of elastic scattering at energies above 160 MeV.

3.2 Introduction of the symmetry term

In the intermediate energy region, the potential is based on the sum of interactions between incident nucleon and individual nucleons in the nucleus. From this point of view, the optical model potential consists of potentials of proton-proton, proton-neutron and neutron-neutron. The symmetry of nuclear force on isospin gives the equivalence of interactions of proton-proton and neutron-neutron.

When a proton is incident on a target nucleus of mass number *A*, atomic number *Z* and number of neutrons *N*, a local value of potential *V* has the following dependence in a rough approximation.

$$V \propto \frac{Z}{A}V_{pp} + \frac{N}{A}V_{pn} = \frac{V_{pp} + V_{pn}}{2} + \frac{V_{pp} - V_{pn}}{2}\frac{N - Z}{A}.$$

The first term in the right hand side is the average term, and the second the symmetry one. When a neutron comes into the same nucleus, the potential coincides with the proton potential having the changed sign of the symmetry term. As far as this view holds good, therefore, the proton-incident potentials are converted into the neutron-incident ones by the sign change of the symmetry term[3].

The global parametrization of fit 1 by Cooper et al.[2] is utilized as proton-incident potentials in this study. The potentials are given by functions of the mass number *A* and the incident kinetic energy *T*, but do not include the symmetry term that is dependent on *(N-Z)/A*. As a first approach, we take

notice of the data on ^{90}Zr and ^{208}Pb among the nuclei used in evaluation of the potentials. The symmetry term was simply derived from the potentials on the two nuclei, on the basis of its expression of $b(N-Z)/A$. The values of b are listed in Table 1 at some kinetic energies. To obtain the neutron-incident potentials, two-fold of the symmetry term is subtracted from the proton-incident potential: This is equivalent to the change of the sign of the symmetry term in the proton-incident potentials. The Coulomb potential V_C vanishes in the neutron incidence in the nature of Coulomb interaction.

Table 1 Values of the symmetry term

T(MeV)	b_{VV}	b_{WV}	b_{VS}	b_{WS}	b_{WVSP}	b_{WSSP}
50	188	84.7	-280	-118	4.69	-4.41
100	188	84.5	-280.7	-118	4.10	-4.03
200	185	89.7	-279	-123	-0.587	-0.346
300	179	93.5	-272	-127	-3.35	1.89
400	173	94.7	-265	-131	-3.36	2.68
500	169	94.9	-260	-135	-1.86	3.53
700	175	97.52	-267	-153	1.12	9.64
1000	213	114.7	-318	-205	-2.10	34.1

The total cross sections for neutron incidence are calculated by the potentials. The results are plotted by dashed lines in Fig. 1, where dotted ones stand for the calculation as a reference by the use of original proton-incident potentials after eliminating the Coulomb potential. Although the original parametrization

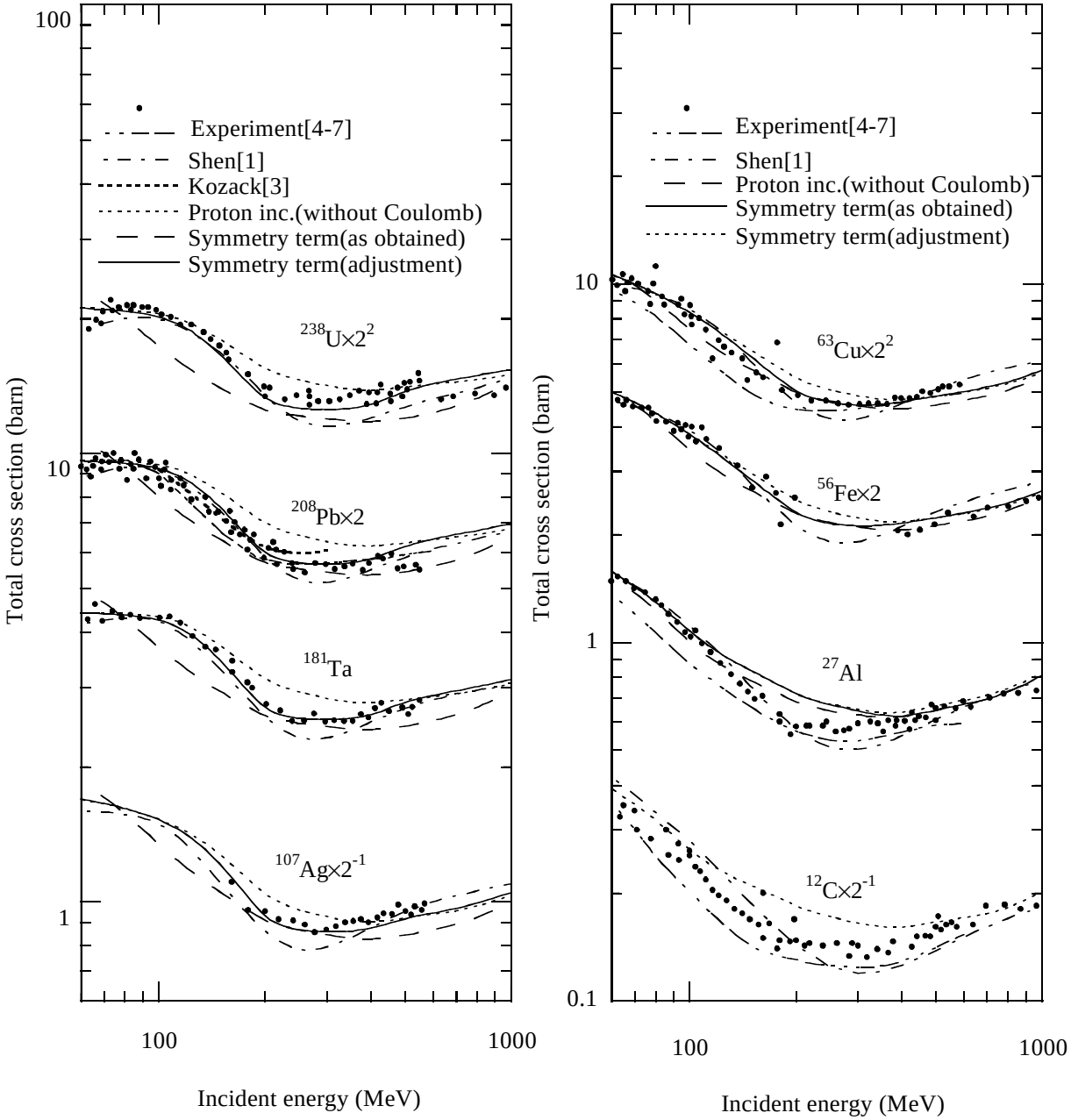


Fig.1 Neutron total cross section

by Cooper et al. covers nuclei up to ^{208}Pb, cross sections are calculated for ^{238}U beyond the coverage for comparison. One can see that the dashed lines are in good agreement with the experimental data for ^{56}Fe to ^{208}Pb at energies of 200 to 300 MeV. It is quite interesting that the simple estimation of symmetry term based on the difference of potentials between ^{90}Zr and ^{208}Pb holds good in this energy region. The dashed lines underestimate the experimental data in the energy region of 80 to 200 MeV, and also in the range above 300 MeV.

To find the influence of the symmetry term, a quantity of $f = (\ _1 - \ _0)/(\ _e - \ _0)$ is evaluated, where $_0$ is the calculated cross section for incident protons without the Coulomb potential, $_e$ the experimental cross sections and $_1$ the calculated cross section after sign change of the symmetry term (neutron incidence). The results are plotted in Fig. 2 with dashed lines. The values of f are almost unity mainly at energies of 200 to 300 MeV. In contrast, f is lower than unity out of this region, that is, the symmetry term is overestimated. There is a tendency for the energy region of pertinent estimation of symmetry term to get slightly wider as the mass number A decreases.

The values of f are fitted by a smooth function as

$$f = \frac{1}{1+e^{-(T-T_1)/a_1}} \frac{1}{1+e^{(T-T_2)/a_2}},$$

where T is the kinetic energy of incident nucleons, and quantities such as T_1, T_2, a_1 and a_2 are adjustable parameters. The parameters are listed in Table 2, and the values of f are plotted in Fig. 2 by solid lines. The use of f is considered as a measure of correction for $_1$ to reach $_0$. As a second approach, therefore, the symmetry term is adjusted by multiplying the first-estimated values by f. The solid lines in Fig. 1 indicate the calculated cross section by the use of symmetry term after this adjustment. For heavy nuclei like ^{181}Ta and lighter ^{56}Fe, the solid lines reproduce the experimental data completely in the whole energy region, and give better results than chain lines obtained from the potentials by Shen et al. For ^{107}Ag and ^{63}Cu, the solid lines represent better the experimental data than the chain ones in the energy range below 400 MeV, but underestimate the data above this energy. The behavior of cross sections of ^{107}Ag and ^{63}Cu above 400 MeV seems to be different in quality from those of ^{181}Ta and ^{56}Fe.

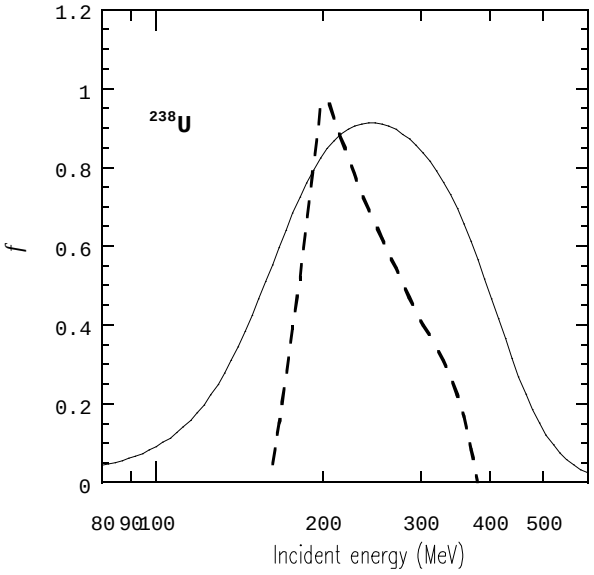
Table 2 Parameters for the adjustment factor f. The parameters

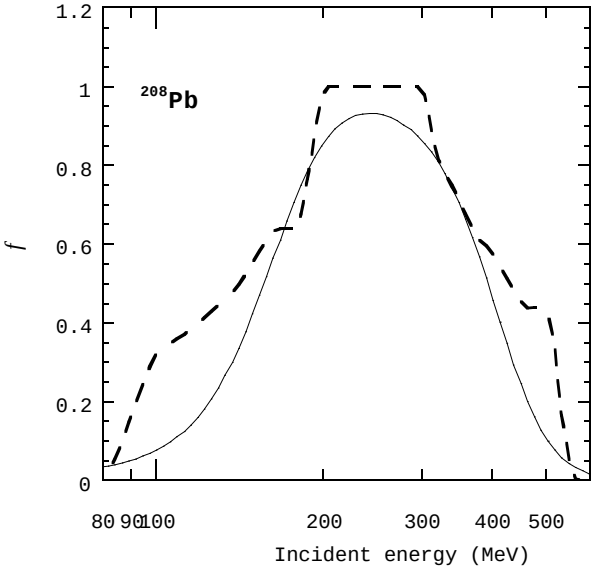
	T_1	a_1	T_2	a_2
c	152	-4.42	379	-17.2
c	20.5	127	72.6	322

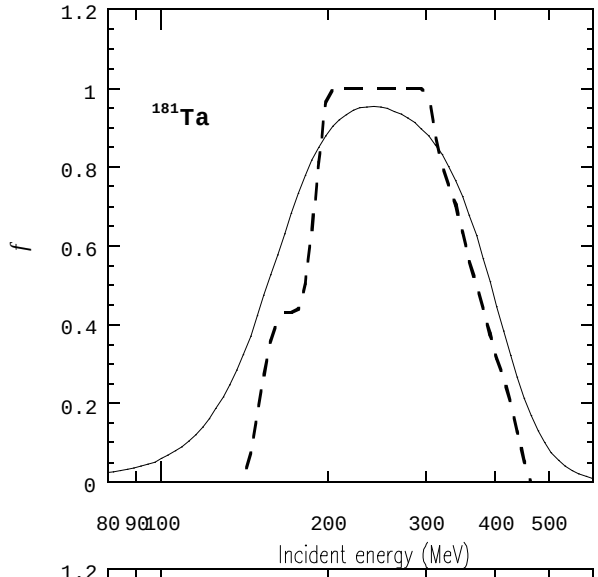
are expressed in a form of $c_1+c_2(N-Z)/A$.

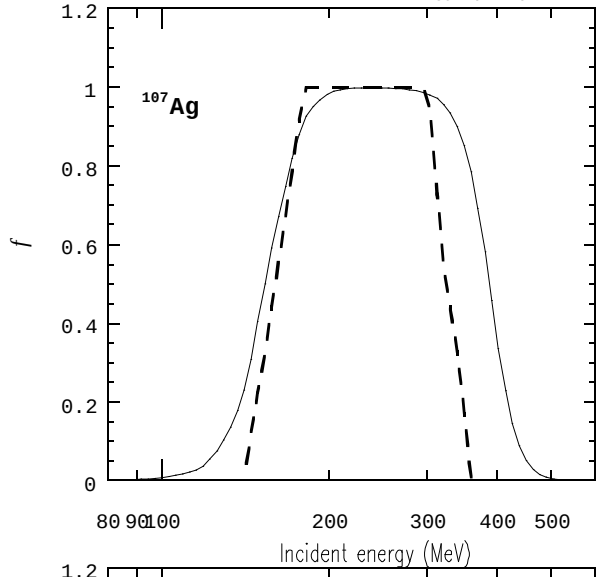
4. Neutron-incident differential cross sections

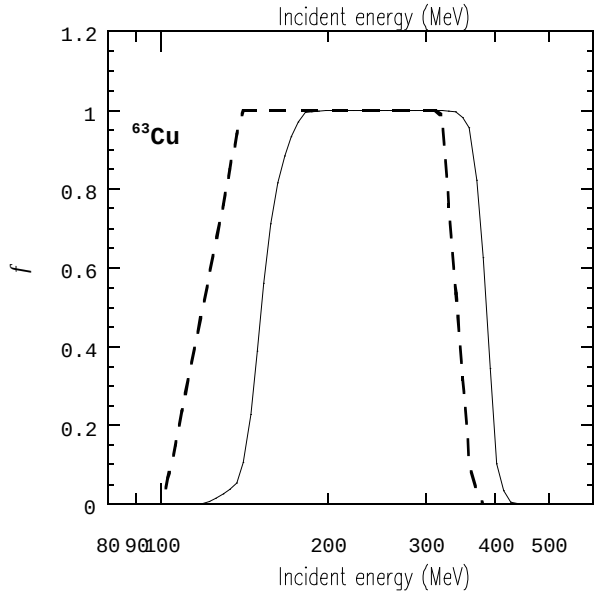
Neutron incident differential cross sections for the elastic scattering are calculated by the use of potentials obtained above. The results are plotted by solid lines in Fig. 3. The solid line for ^{63}Cu is almost the same as the chain line by the potentials of Shen et al. For ^{208}Pb, the solid line gives similar results to the dotted one of Kozack. at angles below 9º, whereas the solid line deviates from the lines of Kozack et al.exponential type parametttization and Shen et al. The deviation may come mainly from invalidity of the original global parametrization of Cooper et al. for proton incidence on ^{208}Pb.

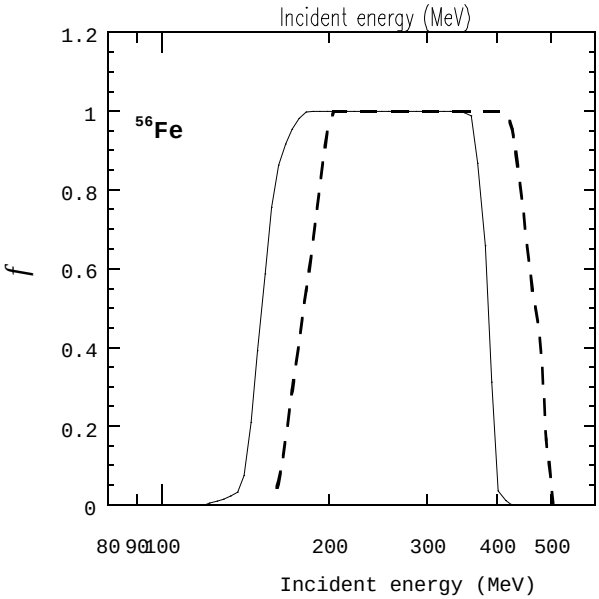

^{238}U
f
Incident energy (MeV)


^{208}Pb
f
Incident energy (MeV)


^{181}Ta
f
Incident energy (MeV)


^{107}Ag
f
Incident energy (MeV)


^{63}Cu
f
Incident energy (MeV)


^{56}Fe
f
Incident energy (MeV)

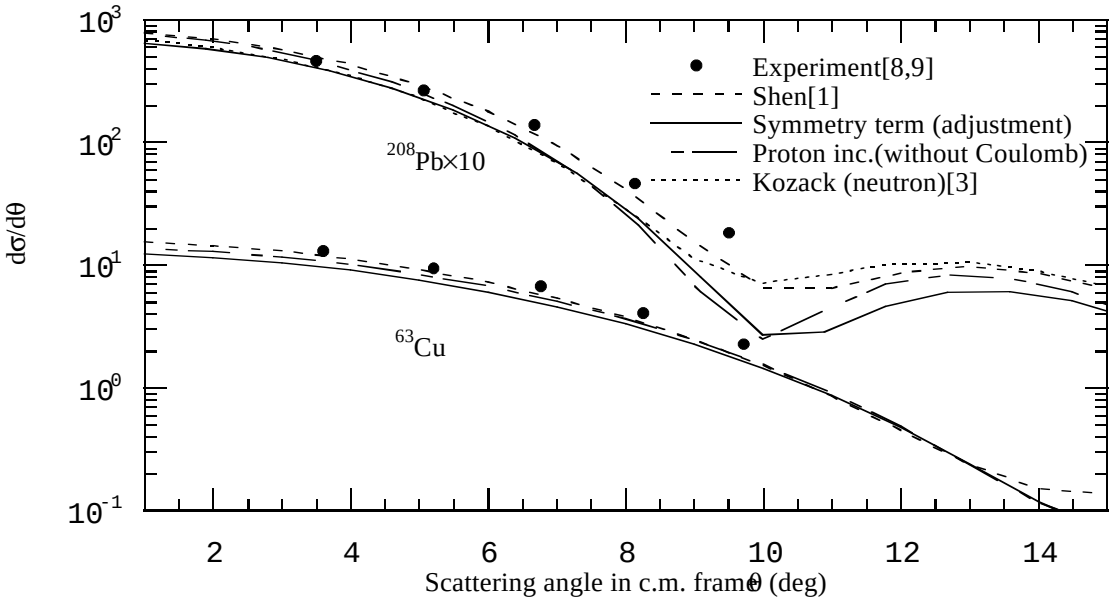


Fig.3 Neutron-incident differential cross section at T=155MeV

5. Comparison between potentials

Present potentials for ^{208}Pb are plotted in Fig. 4 together with those of Kozack. and Shen et al. The potentials are displayed in a form of volume integral for proton and neutron incidence, to make clear the size of the symmetry terms. Although Shen et al. parametrized the potential parameters for neutrons with symmetry terms, the potentials for protons are simply deduced by changing the sign of symmetry terms. For U_{VR} at an energy of 200 MeV, the solid line for proton incidence in the present parametrization decreases to the chain line for neutron, i.e. a positive value of b in the symmetry term. In contrast, potentials by Shen et al. has no change in U_{VR} for proton and neutron incidence, and the dashed line for proton incidence in Kozack. increases to the dotted line for neutron, corresponding to a negative value of b. For U_{VI}, both present parametrization and Shen et al. have positive values of b, whereas that of Kozack owns a negative quantity. A similar contradiction appears in U_{SR} and U_{SI}. Although the sign of symmetry term is based on dependence of proton incidence parameters on target nuclei, it brings about a confusion in the sign of symmetry term. More studies are required to clear up the confused situation.

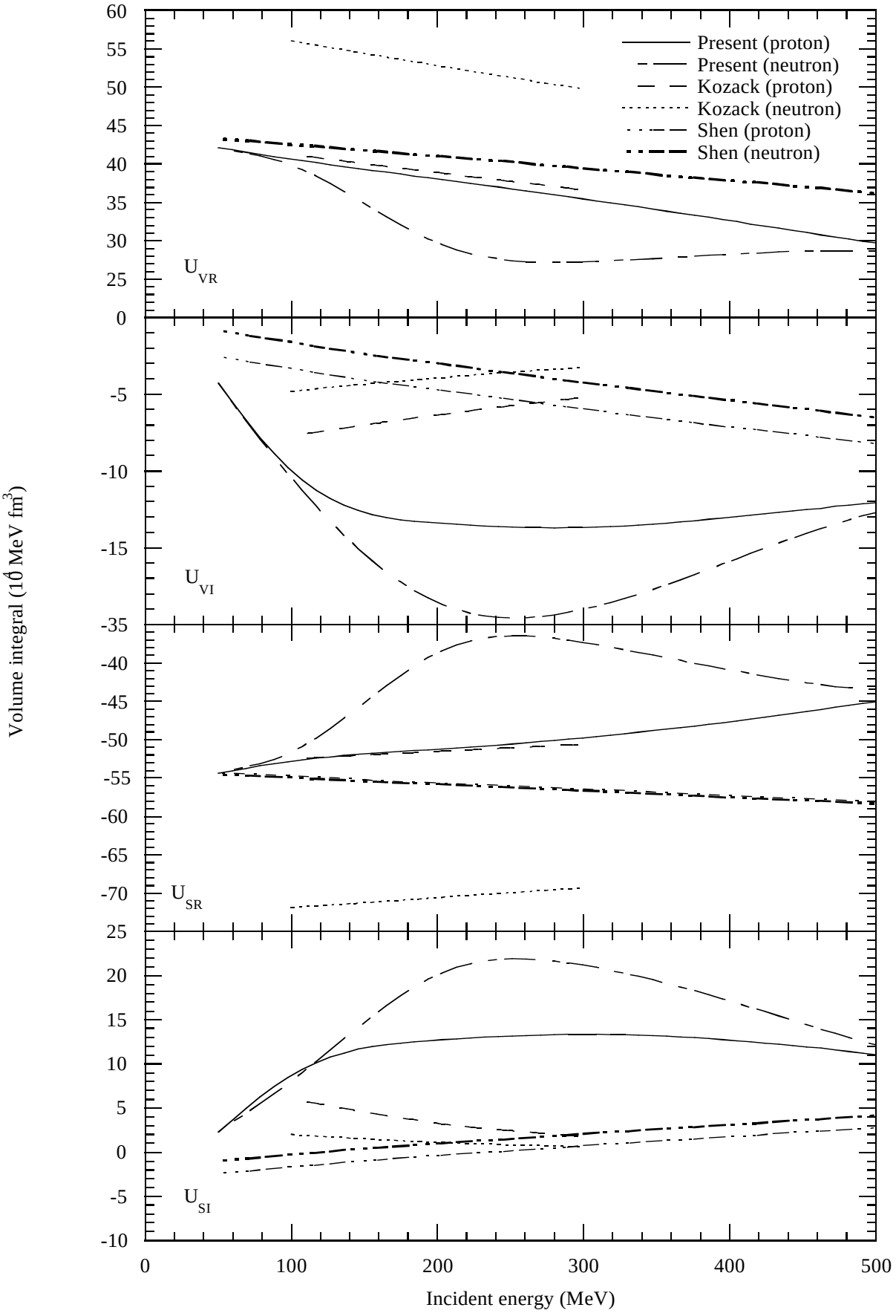


Fig.4 Volume Integral for ^{208}Pb

6. Concluding remarks

Neutron-incident optical model potentials were obtained by introduction of the symmetry term into proton-incident global potentials. The symmetry term shows a special tendency as a function of the incident energy, and its behavior in the energy region of 200 to 300 MeV is different from that out of the region. The potentials obtained for neutron incidence reproduce the total cross sections for Fe or heavier nuclei at neutron energies of 100 to 300 MeV. The calculated neutron-incident differential cross sections for elastic scattering are in fair agreement with the experimental data for target nuclei. For potentials proposed by present and other authors, the confused situation remains in the sign of the symmetry terms.

Acknowledgments

The authors express their gratitude to Prof. Masaru Matoba of Kyushu University for his useful discussions.

References

[1] Shen, Q.B., Feng, D.C., and Zhuo, Y.Z.:*Phys. Rev*., **C43**, 2773 (1991).
[2] Cooper, S., et al.:*Phys. Rev.*, **C47**, 297 (1993).
[3] Kozack, R., Madland, D.G.:*Phys. Rev.*, **C41**, 2737 (1990).
[4] Franz, J., et al.: *Nucl. Phys*., **A 490**, 667 (1988), Pearlsteinin, S.:*Nuclear Data for Basic and Applied Science Proceedings of the Internatioanl Conference Santa Fe*,New Mexico.
[5] Mclane, V., Dunford, C.L., and Rose, P.F.:*"Neutron Cross Sections",* Vol. 2.
[6] Schutt, R.L. et al.:*Phys. Lett.*, **B203**, 22 (1988).
[7] Finlay,R.W. et al.:*Phys. Rev.*, **C47**, 237 (1993).
[8] Salmon, G.L.:*Nucl. Phys*., **21**, 15 (1960).
[9] Harding, R.S.:*Phys. Rev*., **111**, 1164 (1958).
[10] The differential cross section and analyzing power data are in Hutcheon, D.A. et al.*Nucl. Phys*., **A 483**, 429 (1988),and the spin-rotation data are in Ma Ji, Masters thesis,Simon Fraser University,1987(unpublished);Ma Ji et al. (unpublished)