CORE REGISTRY
Ne pas prendre S.V.P.
Please do not take

NEA/CSNI/R(94)20/Part.1

OECD

NEA

REPORT OF A CSNI WORKSHOP ON UNCERTAINTY ANALYSIS METHODS

London, 1-4 March 1994

Volume 1

August 1994

COMMITTEE ON THE SAFETY OF NUCLEAR INSTALLATIONS
OECD NUCLEAR ENERGY AGENCY
Le Seine Saint Germain - 12, Boulevard des Iles
F-92130 Issy-les-Moulineaux (France)
Tel. (33-1)45 24 82 00 Fax (33-1)45 24 11 10
Electronic mail : NEA@FRNEAB51

ORGANISATION FOR ECONOMIC CO-OPERATION AND DEVELOPMENT

Pursuant to Article 1 of the Convention signed in Paris on 14th December 1960, and which came into force on 30th September 1961, the Organisation for Economic Co-operation and Development (OECD) shall promote policies designed:

— to achieve the highest sustainable economic growth and employment and a rising standard of living in Member countries, while maintaining financial stability, and thus to contribute to the development of the world economy;
— to contribute to sound economic expansion in Member as well as non-member countries in the process of economic development; and
— to contribute to the expansion of world trade on a multilateral, non-discriminatory basis in accordance with international obligations.

The original Member countries of the OECD are Austria, Belgium,Canada, Denmark, France, Germany, Greece, Iceland, Ireland, Italy, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States. The following countries became Members subsequently through accession at the dates indicated hereafter: Japan (28th April 1964), Finland (28th January 1969), Australia (7th June 1971), New Zealand (29th May 1973) and Mexico (18th May 1994). The Commission of the European Communities takes part in the work of the OECD (Article 13 of the OECD Convention).

NUCLEAR ENERGY AGENCY

The OECD Nuclear Energy Agency (NEA) was established on 1st February 1958 under the name of the OEEC European Nuclear Energy Agency. It received its present designation on 20th April 1972, when Japan became its first non-European full Member. NEA membership today consists of all European Member countries of OECD as well as Australia, Canada, Japan, Republic of Korea, Mexico and the United States. The Commission of the European Communities takes part in the work of the Agency.

The primary objective of NEA is to promote co-operation among the governments of its participating countries in furthering the development of nuclear power as a safe, environmentally acceptable and economic energy source.

This is achieved by:

— *encouraging harmonization of national regulatory policies and practices, with particular reference to the safety of nuclear installations, protection of man against ionising radiation and preservation of the environment, radioactive waste management, and nuclear third party liability and insurance;*
— *assessing the contribution of nuclear power to the overall energy supply by keeping under review the technical and economic aspects of nuclear power growth and forecasting demand and supply for the different phases of the nuclear fuel cycle;*
— *developing exchanges of scientific and technical information particularly through participation in common services;*
— *setting up international research and development programmes and joint undertakings.*

In these and related tasks, NEA works in close collaboration with the International Atomic Energy Agency in Vienna, with which it has concluded a Co-operation Agreement, as well as with other international organisations in the nuclear field.

COMMITTEE ON THE SAFETY OF NUCLEAR INSTALLATIONS

The NEA Committee on the Safety of Nuclear Installations (CSNI) is an international committee made up of scientists and engineers. It was set up in 1973 to develop and coordinate the activities of the OECD Nuclear Energy Agency concerning the technical aspects of the design, construction and operation of the nuclear installations insofar as they affect the safety of such installations. The Committee's purpose is to foster international co-operation in nuclear safety amongst the OECD Member countries.

© OECD 1994
Applications for permission to reproduce or translate all or part
of this publication should be made to:
Head of Publications Service, OECD
2, rue André-Pascal, 75775 PARIS CEDEX 16, France.

REPORT OF A CSNI WORKSHOP ON

UNCERTAINTY ANALYSIS METHODS

London, 1-4 March 1994

VOLUME 1

A. J. Wickett[1]
Winfrith Technology Centre

and

G. Yadigaroglu[2]
Swiss Federal Institute of Technology, Zurich

August 1994

(1) AEA Technology,
Winfrith Technology Centre,
Dorchester,
Dorset
DT2 8DH
UK

(2) Swiss Federal Institute of Technology,
ETH-Zentrum, CLT,
CH-8092 Zurich,
Switzerland

REPORT OF A CSNI WORKSHOP ON UNCERTAINTY ANALYSIS METHODS

A. J. Wickett
AEA Technology, Winfrith Technology Centre
G. Yadigaroglu
Swiss Federal Institute of Technology, Zurich

August 1994

SUMMARY

The OECD NEA CSNI Principal Working Group 2 (PWG2) Task Group on Thermal Hydraulic System Behaviour (TGTHSB) has, in recent years, received presentations of a variety of different methods to analyze the uncertainty in the calculations of advanced unbiased ("best estimate") codes. At its meeting on 30 June - 2 July 1993 it accepted a UK offer to host a Workshop to discuss the various methods that had been proposed. The PWG2 also asked that the Workshop discuss proposals for an International Standard Problem (ISP) to compare the uncertainty analysis methods.

The objectives for the Workshop, defined at the December 1993 meeting of the TGTHSB were:

- To discuss and fully understand the principles of uncertainty analysis relevant to LOCA modelling and like problems.
- To examine the underlying issues from first principles, in preference to comparing and contrasting the currently proposed methods.
- To reach consensus on the issues identified as far as possible while not avoiding the controversial aspects.
- To identify as clearly as possible unreconciled differences.
- To issue a "Status Report".

The Workshop was held on 1-4 March 1994 at the Strand Palace Hotel, London. A total of 30 delegates from 12 countries attended. Professor G. Yadigaroglu, of the Swiss Federal Institute of Technology, Zurich, chaired the meeting on 1-3 March. Eight uncertainty analysis methods were presented. A structured discussion of various aspects of uncertainty analysis followed - the need for uncertainty analysis, identification and ranking of uncertainties, characterisation, quantification and combination of uncertainties and applications, resources and future developments. As a result, the objectives set out above were, to a very large extent, achieved.

On 4 March plans for the ISP were discussed. Mr S. N. Aksan, Chairman of the TGTHSB, chaired this part of the Workshop. The objectives for the ISP were redefined. A shortlist of two experiments was drawn up.

Volume 1 of this report contains the record of the discussions and conclusions. Volume 2 contains accounts of the uncertainty analysis methods presented, provided by the authors.

Further copies of either volume of the report may be obtained from the OECD Secretariat or from A. J. Wickett at the address on page i.

CONTENTS LIST

		Page
Title Page		i
Summary		iii
Contents		v
1.	Introduction	1
1.1	Objectives of the Workshop	1
1.2	Organisation of the Workshop	1
1.3	Organisation of the Workshop Report	2
2.	Methods Presented	3
3.	The Need for Uncertainty Analysis	5
4.	Identification and Ranking of Uncertainties	9
4.1	Identification	9
4.2	Ranking	9
5.	Characterisation of Uncertainties	11
6.	Quantification of Uncertainties	13
7.	Combination of Uncertainties	15
7.1	Methods and their justification	15
7.2	Scaling	15
7.3	Compensating errors	16
7.4	Bifurcations	16
7.5	The use of judgement	16
7.6	Probability and Confidence Statements	16
8.	Applications	17
9.	Resources (Staff Time and Computer Time)	19
10.	Future Developments	21

11. Conclusions 23

11.1 Principles of Uncertainty Analysis 23

11.1.1 Need for Uncertainty Analysis (Section 3) 23
11.1.2 Identification and Ranking of Uncertainties (Section 4) 23
11.1.3 Characterisation and Quantification of Uncertainties (Sections 5 and 6 and Appendix 4) 23
11.1.4 Combination of Uncertainties (Section 7) 24
11.1.5 Applications, Resources and Future Developments (Sections 8, 9 and 10) 24

11.2 Overall Conclusions and Recommendations 26

12. References 27

Appendix 1: Attendance List 29

Appendix 2: Agenda 31

Appendix 3: Proposed International Standard Problem 35

Appendix 4: Arguments for and against the use of Subjective Probability Distributions 37

OECD NEA CSNI PWG2
Task Group on Thermal-Hydraulic System Behaviour

SPECIAL WORKSHOP ON UNCERTAINTY ANALYSIS METHODS

London, 1-4 March 1994

1. Introduction

1.1. Objectives of the Workshop

The OECD NEA CSNI Principal Working Group 2 (PWG2) Task Group on Thermal Hydraulic System Behaviour (TGTHSB) has, in recent years, received presentations of a variety of different methods to analyze the uncertainty in the calculations of advanced unbiased ("best estimate") codes. At its meeting on 30 June - 2 July 1993 it accepted a UK offer to host a Workshop to discuss the various methods that had been proposed. The PWG2 also asked that the Workshop discuss proposals for an International Standard Problem (ISP) to compare the uncertainty analysis methods.

The objectives for the Workshop were defined, at the December 1993 meeting of the TGTHSB, to be:

- To discuss and fully understand the principles of uncertainty analysis relevant to LOCA modelling and like problems.
- To examine the underlying issues from first principles, in preference to comparing and contrasting the currently proposed methods.
- To reach consensus on the issues identified as far as possible while not avoiding the controversial aspects.
- To identify as clearly as possible unreconciled differences.
- To issue a "Status Report".

1.2 Organisation of the Workshop

The Workshop was held on 1-4 March 1994 at the Strand Palace Hotel, London. A total of 30 delegates from 12 countries attended. The attendance list is given in Appendix 1 of this Report. The Agenda is given in Appendix 2 of this Report.

Professor G. Yadigaroglu, of the Swiss Federal Institute of Technology, Zurich, chaired the meeting on 1-3 March. Eight uncertainty analysis methods were presented. The speakers are listed in Table 1 (on the next page). Each speaker was asked to address a number of aspects of their method. These are listed in the Agenda, Appendix 2. A number of questions were then formulated and they were discussed on the second day. Agreement was attained on many, but not all, of the issues discussed. The objectives set out above were, to a very large extent, achieved.

On 4 March plans for the ISP were discussed. Mr N. Aksan, Chairman of the TGTHSB, chaired this part of the Workshop. The objectives for the ISP were redefined. A shortlist of two experiments was drawn up.

Table 1: Methods Presented and Main Speakers

CSAU Method (USA)	Gary Wilson, INEL
UMAE Method (Italy)	Francesco D'Auria, Pisa
AEA Method (UK)	Tony Wickett, AEA Technology
NE Method (UK)	Martin Trow, NNC
GRS Method (Germany)	Horst Glaeser, GRS
CEA Method (France)	Raymond Pochard, IPSN, et al
Tractebel Method (Belgium)	Luc Vanhoenacker, Tractebel
Limit Value Approach (USA)	Ed Lee, ABB-CE

1.3 Organisation of the Workshop Report

The Workshop Report consists of two volumes. Volume 1 contains a record of the discussions on uncertainty methods held on 1 to 3 March. A draft report was prepared at the Workshop for agreement by the delegates. The resulting document was presented to the participants in Reference 1. After further editing and the addition of some extra material, it now forms Sections 3 to 10 and 11.2 of Volume 1 of this report.

It is arranged according to a number of important features of the methods so that information about all of them can be combined and compared. It identifies areas of agreement and disagreement and questions that need further work and attempts to answer some basic questions about the limits of code uncertainty estimation. A summary of this discussion has been added at Section 11.1.

Volume 1 also contains Appendix 1 giving the attendance list and Appendix 2, the Workshop agenda. The record of the discussion on 4 March on the proposed uncertainty International Standard Problem (ISP) is given in Appendix 3. This has been agreed by the participants. Appendix 4 gives arguments in favour of and against the uses of subjective probability distributions in uncertainty analysis. This was the main point of disagreement at the Workshop.

Volume 2 is a compilation of descriptions of the eight uncertainty analysis methods presented at the workshop. This has been prepared from material submitted by the authors and has not been edited in any way.

2 Methods Presented

The Workshop opened with presentations of eight different uncertainty analysis methods. The methods were designed to investigate the uncertainty in the prediction of large "best estimate" codes modelling nuclear power plant accidents. Most, but not all, of the methods presented were primarily aimed at LOCA transients. Descriptions of these methods, provided by the presenters, are provided in Volume 2 of this report.

Since one of the objectives of the Workshop was to examine the underlying issues from first principles, rather than comparing and contrasting the methods, only the briefest description of each is given here.

CSAU (Code Scaling, Applicability and Uncertainty) [3, 4] successfully demonstrated one method of quantifying uncertainties. In this demonstration the uncertainty space was sampled with a number of code runs. The code predictions were used to set up a polynomial fitted response surface over the uncertainty space. A subjective probability density function was also set up over the uncertainty space and the response surface sampled according to this distribution. The resulting uncertainty statement for the predicted quantity (peak clad temperature) was modified by biases to account for effects not taken into account in the analysis. This was the first demonstration of an uncertainty study for large LOCA predictions with an unbiased code and as such was a very interesting exercise. This is evidenced by the extensive discussion that followed [12].

The UMAE (Uncertainty Methodology based on Accuracy Extrapolation) is based upon the extrapolation of the accuracy of calculations of integral experiments from test scale to plant scale. This relies upon the identification of "counterpart" or similar tests in which the thermalhydraulic phenomena are similar to those expected in the reactor transient. Required conditions for accuracy extrapolation include that relevant thermalhydraulic aspects are the same in the experiments and the plant; that they are quantitatively well predicted by the code for the experiments; that appropriate scaling can be identified; that relevant parameters, dimensionless numbers and time sequences can all be scaled at the same time and, when scaled, all these relevant parameter ranges are the same in the experiments and the plant.

The two UK methods, **the AEA and NE (Nuclear Electric) methods**, sample within the uncertainty space with a number of code runs. Physical and mathematical understanding of the processes taking place and their interactions is used to define further runs with the aim of identifying the entire range of the output quantity of interest. In the NE method some biases were also added afterwards as in the CSAU demonstration.

The GRS method defines subjective probability density functions for all identified potentially important uncertainties. By performing a specified number of code calculations at random according to these distributions desired percentiles (such as the 95th) of the corresponding subjective probability distributions for the predicted output quantities of interest can be estimated. Additional uncertainty due to approximation by a response surface is avoided.

The CEA method is still under development. The sensitivity of the output quantity of interest to each of the uncertain parameters is calculated throughout the calculation either by

sampling in the uncertainty space and using multiple linear regression analysis or by the adjoint sensitivity method which relies on the code to calculate partial derivatives of the output quantities of interest with respect to the uncertain parameters at each timestep. Either method leads to a response surface approximation to the dependency of the quantity of interest on the uncertain parameters. This is then sampled at random according to a subjective probability distribution assigned over the uncertainty space. As in the GRS method, a predetermined number of sample points allows the upper 95th (etc) percentile of the corresponding subjective probability distribution of the output quantity of interest to be estimated.

Limit Value Methods from Tractebel and ABB-CE were presented. The Tractebel method was demonstrated for a feedwater line break. Initial and boundary conditions were set to the most adverse for the selected key parameter (minimum collapsed core level). The effects of other uncertainties were separately investigated. Only the resulting negative biases were then added together to be subtracted from the reference calculation value to give the licensing value for the key parameter. The ABB-CE method evaluates penalties in the key parameter (peak clad temperature, PCT) separately due to code uncertainty, plant parameter uncertainty and application uncertainty and combines them by adding. As in the Tractebel method, the plant parameter penalty is obtained from the set of conditions which will cause the worst PCT.

3 The Need for Uncertainty Analysis

Three main reasons were identified:

Licensing and safety - the move from licensing based on evaluation models to the use of "best estimate" calculations with uncertainty estimates. This was the prime motivation for development of uncertainty analysis methods. In addition uncertainty analysis allows more realistic estimates of the safety margins of nuclear power plants.

Accident Management - it was agreed that analyses in support of accident management should include uncertainty estimates, but that this did not mean that uncertainty estimates should be given directly to the operators in guidance. The methods provided potential for improvement of Emergency Response Guidelines (ERGs) but the expectations were modest. The aim of uncertainty analysis in this area should be to ensure that recommendations are reasonable and to pinpoint areas where more detailed knowledge is needed to define improved ERGs.

Research prioritisation - it was agreed that uncertainty analysis could help identify correlations and code models that needed the most improvement. It can also identify areas where more data are needed. It could make code development and validation more cost-effective. It helped to evaluate and to improve the quality of a computer code. It can be used to define the point at which a code has been sufficiently developed. It can also be used to diagnose whether new versions are real improvements.

Different licensing authorities have different requirements and this would influence the nature and extent of uncertainty analysis needed to satisfy them.

The requirements for the inclusion of uncertainty analysis for licensing in each country and status of the applications is summarised in Table 2 (on the next page). The TGTHSB has undertaken a more comprehensive review of this question. A status report on the application of best estimate codes in licensing is being prepared [2].

Table 2:
Licensing Requirements for LOCA Uncertainty Analysis and Status in each Country

Country	Requirement	Status
Belgium	Uncertainty method described at the Workshop is not applied to LOCA.	Bounding, limit value approach has been used with RELAP-5/MOD2 to assess uncertainty in non-LOCA transients. Method was acceptable to allow power uprating of 10% for Doel-3 PWR.
France	Not discussed at the Workshop	CEA/IPSN method under development.
Germany	Uncertainty analysis not mandatory.	Best estimate calculations with conservative initial and boundary conditions have been accepted. GRS uncertainty analysis method being developed and demonstrated. Can provide uncertainty statement for time-dependent parameters.
Italy	Evaluation Method has been adopted in Italy up to the last plant, licensed in 1986. However best estimate calculations have been used extensively to evaluate the performance of ECCS and related safety margins. Uncertainty analysis is not required.	Uncertainty analysis methods are under consideration, but no specific choice has been made yet. UMAE method could represent a way for best estimate code calculation uncertainty.
UK	Licensing framework is not prescriptive.	Nuclear Electric chose licensing methodology based on physically based mechanistic codes including treatment of uncertainties. Case has been submitted to NII and accepted in principle, see Volume 2.

Country	Requirement	Status
USA	Can either use Appendix K Model without uncertainty analysis or a realistic evaluation model with quantification of uncertainty to show that there is a high probability that criteria will not be exceeded (10CFR50.46 and Appendix K, amended in 1988).	CSAU process designed to demonstrate use of amended regulations, demonstrated for LB LOCA and SB LOCA by USNRC contractors. Similar methods applied by GE and Westinghouse. Limit Value Approach developed by ABB-CE, submitted to USNRC in 1993, being reviewed.

4 Identification and Ranking of Uncertainties

4.1 Identification

Table 3 provides a list of various sources of uncertainty. Some of these may not be treated in some studies. It was agreed that it does not matter how uncertainties are classified, but the process of classification can be helpful in ensuring that none have been left out. It may be useful to classify uncertainties by how they will be treated in the analysis.

Table 3: Sources of Uncertainty

Conservation equations
Numerical representation and solution of conservation equations
All phenomena modelled?
Models and correlations
Geometrical presentation (nodalisation)
Initial conditions
Boundary conditions
Operator actions
Scaling effects
Programming errors
Input deck errors
Other user effects
Computer and compiler effects
Material properties
Adequacy and quality of experimental data base

The meeting discussed whether one should separate regulatory assumptions (such as initial power level, peaking factor etc) from code-related uncertainties stemming from imperfect modelling. It was agreed that the analyst should distinguish between the two, so as to report the results more clearly.

Uncertainty in operator actions (such as delay in actuating a system) should be treated as with other uncertainties.

4.2 Ranking

It was agreed that the use of Phenomena Identification and Ranking Tables (PIRT) [3, 4], based on collective subjective judgement, was an acceptable method for identification of important uncertainties, but it was also recognised that the need for PIRTs was reduced when a cost-effective method that includes all identified sources of uncertainty was used.

All analyses had shown some surprises, identifying levels of contribution to uncertainty different from those in the PIRT. Therefore it would be useful to contrast PIRTs with the subsequent analyses. If a PIRT is used to eliminate minor sources of uncertainty, there should be a check that combinations of those ignored are still unimportant. Large break LOCA reflood sensitivity analysis gave results with some very high and non-monotonic sensitivities detected. This calls for caution when judging uncertainties to be unimportant.

The effort for alternative, more rigorous or mathematical approaches proposed in Germany and France does not appear to require correspondingly increased resources. Because there is no restriction on the number of uncertain parameters, these methods have an additional benefit of increased comprehensiveness in the stated uncertainty.

The French Adjoint Sensitivity Method (ASM) calculates the local (at the selected point in the uncertainty space) derivative of the response with respect to all the parameters at each time step. This means that there can be a "continuous" local estimation of the sensitivities to each uncertain parameter. These sensitivity estimates can be combined into uncertainty estimates, subject to an assumption of linearity. They are therefore valid for small deviations from the reference value. This could be used for ranking of uncertain parameters, subject to the above restrictions.

5 Characterisation of Uncertainties

There was a full discussion of the relative merits of characterising uncertainties by subjective probability distributions or uncertainty ranges. Both approaches have been used. It was agreed that it was necessary to use all the relevant information available to provide an adequate quantitative representation of the state of knowledge in a form appropriate in each country.

The majority view was that both approaches could be used. The UK, in the minority, argued that subjective probability distributions should not be used. The views on both sides are presented in Appendix 4.

6 Quantification of Uncertainties

In discussing how judgement should be used in quantifying uncertainties, it was agreed that the use of judgement should be replaced to the extent made possible by the development of more objective methods. For example, some correlation parameters cannot be directly measured. Some methods, including the French ASM method and the AEA method, use appropriate separate effects experimental data to determine the range of these parameters.

Adequacy and quality of the available experimental data base must be taken into account when quantifying uncertainties. This step helps to identify any serious deficiencies. These should be reported at an early stage and taken into account when assigning input uncertainties. Generally a deficiency in the data base should lead to a wider uncertainty range than adequate data.

There was a full discussion of how methods should treat numerical effects, including those relating to space noding and timestep. It was agreed that it is necessary to arrive at a mature nodalisation for the plant with sufficient quality assurance even though a very significant effort may be needed. Even when this had been achieved, there may be some remaining sensitivity of the results to nodalisation. This should be treated as an uncertainty.

Codes should be clearly mature enough to include all models needed to address the physical phenomena expected. If there are any missing parts of models that are of second order these can be treated as uncertainties.

7 Combination of Uncertainties

7.1 Methods and their justification

It was noted that there are broadly two types of methods proposed. These were the statistical approaches used in the first USNRC CSAU demonstration and in the French-German approach and the various bounding approaches which have been offered by the UK and some vendors and licensees.

It was agreed that uncertainties that are not statistical (eg the use of algebraic addition of output biases) should not be combined statistically.

There was a discussion of "biases" (as defined in the CSAU demonstration [3, 4]). It was agreed that it was preferable (but not mandatory) to correct code models by, for example, adding on or multiplying by a corrective term to eliminate the need to apply a bias after the calculation. The uncertainty in the corrective term should also be allowed for. This makes the method more uniform and therefore easier to defend. If a bias is applied to the output, then it must be justified as bounding.

It was argued that bounding analyses can provide good insight into sensitivity to key parameters. However, it was also argued that regression techniques and ASM could provide comprehensive sensitivity information.

7.2 Scaling

The question of scaling was discussed. There are cases when the physical laws are well understood; in such cases the codes are expected to extrapolate to full-scale plant conditions adequately.

Experiments are designed with a scaling rationale. Such experiments can provide an adequate basis for code validation and then extrapolation, provided that the phenomena follow the scaling rationale. If this is not the case, one must use judgement in deciding how to treat the effects of scale upon these phenomena. For example, UPTF revealed unexpected behaviour at full scale. The possibility of unexpected behaviour at full scale needs to be considered as a residual contribution to uncertainty.

The ECCS State of the Art Report gives additional discussion of scale effects [5].

It was agreed that the Pisa method is an interesting, alternative, independently developed method. It will be interesting to compare results.

The uncertainty analysis methods should be applicable if no changes in the relative importance of phenomena or unexpected behaviour occur in uncharted territory. Such possibilities need to be considered as a residual contribution to uncertainty. Uncertainty analysis could be used to define where model development is needed.

7.3 Compensating errors

There was a discussion of compensating errors. It was agreed that code validation should be the main tool for detecting compensating errors. This normally includes tests against experiments that were not used in the code development and the associated developmental assessment. More than one code output and intermediate code results should be tested against data.

The output from uncertainty and sensitivity analysis can suggest the presence of compensating errors if discrepancies are observed between uncertainties and sensitivities to particular uncertain parameters as well as in the uncertainty of final and of intermediate results.

7.4 Bifurcations

It was agreed that the term "bifurcation" was to be preferred to "cliff edge".

Proper modelling of bifurcations should be ensured by code validation. The effects of bifurcations on uncertainty analysis will be naturally accounted for by the methods proposed provided that the input uncertainty ranges are sufficient to encompass all bifurcations and there is sufficient sampling to detect them.

The effects of bifurcations may be smeared by a response surface approximation [6, 7]. The authors of the CSAU demonstration study did not find this to be a problem [8].

7.5 The use of judgement

In discussing the use of judgement, it was agreed that one should continue to reduce reliance on judgement to the maximum extent that is cost effective. The need cannot be completely eliminated.

7.6 Probability and Confidence Statements

It was noted that expressing the result of an uncertainty analysis by a formal confidence level was not currently required for licensing. The revised US licensing requirements (10CFR50.46) require a probability statement. Some statistical methods (GRS method) require the analyst to specify the confidence level; this then determines the minimum number of calculations. In other methods, the number of calculations relies on judgements and the number of sources of uncertainty (eg AEA, CSAU).

It was agreed that the value of uncertainty analysis results, expressed as an uncertainty statement, is significantly enhanced when measures of sensitivity are also given.

8 Applications

Although USNRC SAR Chapter 15 or equivalent requirements specify the analysis of a large number of transients, licensees typically concentrate, for a given design, on a small number of limiting accidents. No organisation has applied uncertainty analysis to the full set of SAR analyses.

Examples of possible future applications include:

a) the use of judgement to choose a "nucleus" set of transients and use phenomenological arguments to say that they provide a bounding envelope to the uncertainty analyses and

b) production of one or more composite PIRTs covering the ranges of accidents considered.

The use of the catalogue of transients prepared for accident management purposes may be useful [11]. The utility of such approaches (a and b above) remain to be demonstrated and such efforts are under way.

It was suggested that only a small number of uncertainty analyses could provide generic estimates of uncertainty for classes of plants, accidents and codes which could then be used for future purposes. Then new designs will still require new studies.

It can be argued that every "best estimate" calculation must be supplemented by its own uncertainty analysis if the results are to be used to support sufficiently important decisions.

9 Resources (Staff Time and Computer Time)

It was felt that as more experience was gained, the cost of uncertainty analyses would go down, especially if judgement (such as PIRTs) is replaced by methods that can treat all uncertainties together, combined with information on uncertain parameters stored in a data base.

The following data on resources used or estimated (est) to be needed were taken from the presentations to the Workshop or subsequently provided by participants. A blank entry indicates that the information was not given.

Table 4: Resources

Method	Test (SET or IET) or plant	Development		Demonstration		Production	
		Person-years	Computer runs	Person-years	Computer runs	Person-Years	Computer runs
CSAU	Plant	9		4	8	2 - 3 est	
UMAE	Plant	5 - 10 est	>100 est	3 - 9 est	10 est	0.1-2 est	1 - 10 est
AEA	IET	3	35				
	Plant			3.5 est	40 est		
NE	Plant				68		29
GRS	SET			1.5	100		
	Plant					7 est	46
CEA	Either	2 - 6 est					
Tractebel	Plant (1)			0.6		0.2	
ABB-CE	Plant						

(1) Non-LBLOCA transients

10 Future Developments

It was suggested that an uncertainty data base should be set up to store information on parameter uncertainties. Although this information is partly code and application dependent, valuable information can be transferred from one study to another.

It is recommended that the TGTHSB should make a proposal for the creation of such a data base.

11 Conclusions

In Section 11.1 we summarise the conclusions arising from the discussion in Sections 3 to 10 above. In Section 11.2 we present the overall conclusions written at the Workshop.

11.1 Principles of Uncertainty Analysis

The main discussion of the Workshop was organised so as to address particular aspects of uncertainty analysis. The conclusions reached are summarised here. References to sections of the report where the discussion is recorded are given.

11.1.1 Need for Uncertainty Analysis (Section 3)

Three reasons were identified:

- To be able to use "best estimate" codes for licensing
- Improvement of Emergency Response Guidelines (modest expectations)
- Research prioritisation.

11.1.2 Identification and Ranking of Uncertainties (Section 4)

It does not matter for an uncertainty analysis how uncertainties are classified (for example into input, modelling, data, numerical etc uncertainties), so long as the list is complete.

Some methods can only treat a few uncertainties. Then collective judgement can be used to select important uncertainties (PIRT). However all analyses had shown some surprises, compared with such judgements. Also some very high and non-monotonic sensitivities have been detected in LOCA reflood analysis. So there should be a check on uncertainties judged unimportant.

Some methods can deal with a bigger number of uncertainties and the problem of selecting important ones is thereby reduced.

11.1.3 Characterisation and Quantification of Uncertainties (Sections 5 and 6 and Appendix 4)

All relevant information should be used to provide an adequate quantitative representation of the state of knowledge. A majority supported the view that it was acceptable to characterise uncertainties either by ranges or by subjective probability distributions. A minority argued that subjective probability distributions should not be used.

The use of judgement in quantifying uncertainties should be replaced as far as possible by more objective methods, for example using appropriate data. Deficiencies in data should lead to wider uncertainty ranges.

Any sensitivity to numerical parameters such as noding should be treated as an uncertainty. Missing parts of models, that are of second order, should be treated as uncertainties.

11.1.4 Combination of Uncertainties (Section 7)

Bounding and statistical approaches (based on subjective probability distributions) were described. Some methods added "biases" after the main calculation. Uncertainty in biases should be allowed for and they should be justified as bounding. It would be inappropriate to combine them statistically.

Unexpected behaviour has sometimes been observed on changes of scale. This must be considered as a contribution to uncertainty.

Modelling experiments that were not used in code development can be used to detect compensating errors, as can uncertainty and sensitivity analysis if intermediate results are examined.

The effects of bifurcations in the models which might lead to discontinuities in the calculated quantities may be smeared if approximated by a response surface.

A common goal of all methods is to reduce the need for subjective judgement as far as practicable - it cannot be completely eliminated. Because of the diversity of the methods presented judgement has to be exercised at different stages and in different ways in each. As an example of this diversity Table 5, on the next page, attempts to summarise how and where judgement is used in some of the methods. A • means that judgement has to be exercised. Two •• mean that judgement has to be exercised to a greater extent than where there is one • in the same horizontal row of the table.

Most, if not all, of the methods described, could also yield information on measures of sensitivity to uncertain quantities. This enhances the value of uncertainty analyses.

11.1.5 Applications, Resources and Future Developments (Sections 8, 9 and 10)

A comparison has been presented of resource requirements of the various methods considered. However much of the information is estimated. Costs are expected to fall, particularly if more uncertainties can be included in studies - cutting the cost of investigating and judging which to include and omit. A large cost is the collection of information on input uncertainties. If this information can be pooled, the cost for each study should go down substantially.

It can be argued that every "best estimate" calculation must have its own uncertainty analysis if the results are to support important decisions.

Future developments may include choosing a "nucleus" set of transients to provide a bounding envelope to the uncertainty analyses.

Table 5: Use of Judgement

	CSAU demonstration [3, 4]	AEA Method	GRS Method	UMAE
Selection of uncertainties for calculation	••	••	•	(1)
Deriving input uncertainty ranges	•	•	•	(1)
Assigning subjective pdfs	•		••	
Selection of points in uncertainty space	•	•		
Deciding when to stop calculations	••	••	•	•
Response surface fitting	•			
Use of biases on output	•			•
Accepting code performance				•
Accepting nodalization				•
Extrapolation (2)				•

(1) Implicit in the choice of experiments (if they are available)

(2) Based only on accuracy extrapolation. Included in uncertainty ranges, subjective pdfs, biases etc in methods other than UMAE.

11.2 Overall Conclusions and Recommendations

The Workshop recommended that the TGTHSB should make a proposal for the creation of a data base on parameter uncertainties.

The conclusions from the Workshop were:

CSAU [3, 4] successfully demonstrated one method of quantifying uncertainties. It aroused considerable interest, as evidenced by the extensive discussion that followed [12]. There have been several other attempts to quantify uncertainties, including the German (GRS), French (CEA/IPSN), UK (AEA and NE) and Italian (UMAE) ones.

A common goal of all methods is to reduce the need for judgement (which is based on a multitude of inputs including technically defensible evidence). Significant progress in replacing judgement in the statistical methods by mathematical treatment has been made in the French and German methods. The Italian method is based on extrapolation. The UK methods are bounding. The diversity of the methods means that judgement is exercised at different stages and in different ways in each, as shown in Table 5 above.

As new improvements are integrated together, we will have technically improved methods. The rate and extent of this progress will depend on cost-effectiveness.

Most studies reported and extensively discussed so far within the CSNI PWG2 TGTHSB were sponsored by governmental or regulatory bodies. Vendors and utilities tend to modify the purely statistical methods, commonly adopting bounding approaches or approaches based on a combination of PIRTs (Phenomena Identification and Ranking Tables, as used in CSAU) and partial statistical treatment and technical justification, based on their perception of cost-effectiveness.

The Workshop was of the opinion that the uncertainty estimation methods considered can provide useful and necessary information about the predictive capabilities of safety analysis codes.

The objectives set for the Workshop have been, to a large extent, achieved.

12 References

1 A. J. Wickett, "Special Workshop on Uncertainty Analysis Methods: Interim Report", AEA RS 5655 (Winfrith, UK, March 1994)

2 D. Bessette, Status Report on the Application of Best Estimate Codes in the Licensing Process of Nuclear Power Plants, in preparation

3 B. Boyack et al, "Quantifying Reactor Safety Margins", NUREG/CR-5249, EGG-2552 (Idaho Falls, ID, December 1989)

4 B. E. Boyack et al, "Quantifying Reactor Safety Margins", six papers in Nuclear Engineering and Design, Vol 119, No 1 (May 1990)

5 M. J. Lewis (ed) et al, "State of the Art Report on Thermalhydraulics of Emergency Core Cooling in Light Water Reactors", OECD NEA CSNI Report no. 161 (Paris, France, October 1989)

6 A. J. Wickett and I. Brittain, "Quantifying Reactor Safety margins: What remains to be done", Nuclear Engineering and Design, Vol 132, 405-408 (January 1992)

7 E. Hofer et al, "Comments on 'Quantifying Reactor Safety Margins'", Nuclear Engineering and Design, Vol 132, 411-414 (January 1992)

8 G. E. Wilson et al, "TPG response to the forgoing letters-to-the-editor", Nuclear Engineering and Design, Vol 132, 431-436 (January 1992)

9 USNRC, "10 CFR Part 50, Emergency core Cooling Systems; Revisions to Acceptance Criteria", US Federal Register, Vol 53, No 180 (September 1988)

10 USNRC, "Best Estimate Calculations of Emergency Core Cooling System Performance", Regulatory Guide 1.157 (March 1989)

11 R. Mandl et al, Accident Management Working Group Final Report, in preparation

12 T. G. Theofanous (ed), eight letters to the editor commenting on Ref 4 and authors' reply, Nuclear Engineering and Design, Vol 132, No 3, pp 403-436 (January 1992)

Appendix 1: Attendance List

Country	Name	Company	Lead Presenter	Tues, 1 Mar	Dinner	Wed, 2 Mar	Thur, 3 Mar	Fri, 4 Mar
Belgium	Mr Callaert	Tractebel		✓	✓	✓	✓	
	Mr Deschutter	Tractebel		✓	✓	✓	✓	
	Mr L. Vanhoenacker	Tractebel	✓	✓	✓	✓	✓	
	Dr Alexandre Babcenco	AV Nucléaire		✓	✓	✓	✓	
Canada	D. J. Richards	AECL		✓	✓	✓	✓	
	Nino Oliva	Ontario Hydro		✓	✓	✓	✓	
France	Mme Agnès de Crécy	CEA		✓	✓	✓	✓	
	M R. Pochard	IPSN	✓	✓	✓	✓	✓	✓
	M A. Ounsy	IPSN		✓	✓	✓	✓	
Germany	Eduard Hofer	GRS		✓	✓	✓		
	Dr Horst Glaeser	GRS	✓	✓	✓	✓	✓	✓
Italy	Prof Francesco D'Auria	University of Pisa	✓	✓	✓	✓		
	Mr M. Leonardi	University of Pisa		✓	✓	✓	✓	✓
	Paolo Marsili	ANPA		✓	✓	✓	✓	
Japan	Tadashi Watanabe	JAERI		✓	✓	✓	✓	
Netherlands	L. Winters	ECN				✓	✓	
Spain	Sr Julio Pérez	CSN		✓	✓	✓	✓	✓
	Sr Juan J. Pena	ENUSA		✓	✓	✓	✓	

Country	Name	Company	Lead Pres-enter	Tues, 1 Mar	Din-ner	Wed, 2 Mar	Thur, 3 Mar	Fri, 4 Mar
	Sr Félix Castillo	Iberdrola		✓	✓	✓	✓	
Sweden	Anders Sjöberg	Studsvik		✓	✓	✓	✓	✓
Switzer-land	Mr Nusret Aksan	PSI		✓	✓	✓	✓	✓
UK	Dr Tony Wickett	AEA	✓	✓	✓	✓	✓	✓
	Dr John Lillington	AEA		✓	✓			✓
	Martin Trow	NNC	✓	✓	✓	✓	✓	
	Peter Lightfoot	Nuclear Electric		✓	✓	✓	✓	
USA	Gary E. Wilson	INEL	✓	✓	✓	✓	✓	✓
	Naugab E. Lee (Ed)	ABB-CE	✓	✓	✓	✓	✓	
EU	Brian Worth	JRC Ispra		✓		✓	✓	✓
OECD	Prof George Yadigaroglu	ETH Zurich		✓	✓	✓	✓	
	Heikki Holmström			✓	✓	✓	✓	✓
	Total		8	29	28	29	27	11

Appendix 2: Agenda

Tuesday, 1 March 1994: Grenville Suite, Strand Palace Hotel

0830	1.	**Welcome and Introductions**	(15 mins)
0845	2.	**Overview of All the Methods - R. Pochard**	(15 mins)
		Presentation of Each Method:	
0900	2.1	**CSAU Method (USA) - Gary Wilson, INEL**	(40 mins)
	2.2	**UMAE Method (Italy) - Francesco D'Auria, Pisa**	(40 mins)
1020	Tea and Coffee		(20 mins)
1040	2.3	**AEA Method (UK) - Tony Wickett**	(40 mins)
	2.4	**NE Method (UK) - Martin Trow**	(40 mins)
1200	Lunch		(80 mins)
1320	2.5	**GRS Method (Germany) - Horst Glaeser**	(40 mins)
	2.6	**CEA Method (France) - R. Pochard et al**	(40 mins)
1440	Tea and Coffee		(20 mins)
1500	2.7	**Tractebel Method (Belgium) - L. Vanhoenacker**	(40 mins)
	2.8	**Limit Value Approach (USA) - Ed Lee, ABB-CE**	(40 mins)
1640	Adjourn		

Wednesday, 2 March 1994: Grenville Suite, Strand Palace Hotel

	3.	**Discussion of aspects of the Methods**	
0830	3.1	**Reasons for studying Uncertainties**	(30 mins)
0900	3.2	**Identification of Uncertainties**	(30 mins)
0930	3.3	**Characterisation of Uncertainties**	(1 hour)
1030	Tea and Coffee		(20 mins)
1050	3.4	**Quantification of Uncertainties**	(1 hour)
1150	Lunch		(80 mins)
1310	3.5	**Combination of Uncertainties**	(2 hours)

1510	Tea and Coffee		(20 mins)
1530	3.6	**Applications**	(30 mins)
1600	3.7	**Resources**	(30 mins)
1630	3.8	**Future Developments**	(30 mins)
1700	Adjourn		

Thursday, 3 March 1994: Grenville Suite, Strand Palace Hotel

0830 4. Discuss and Agree Conclusions and Recommendations of the Workshop

For which aspects are there agreement or acceptance of different approaches offered?
For aspects where there is disagreement, identify as clearly as possible why.
What can be done to resolve or circumvent disagreements?
What further work/writing is needed?
Can we assess the uncertainty in code predictions?
Agree draft report of meeting. (105 mins)

1015	Tea and Coffee	(20 mins)
1035	Continue discussion	(105 mins)
1220	Lunch	(80 mins)
1340	Complete discussion and close	(80 mins)
1500	Tea and Coffee	

Friday, 4 March 1994: AEA London Office, Rex House, 4-12, Regent Street

5. Selection of Experiment for ISP on Uncertainty Analysis Methods

0900	Summary of Aims of ISP - Tony Wickett	(20 mins)
0920	Presentation of proposal - M. Leonardi	(30 mins)
0950	Presentation of proposal - Brian Worth	(30 mins)
1020	Discussion of proposals	(40 mins)
1100	Coffee	
1100	Complete discussion and close	(2 hours)
1300	Lunch	

Discussion of Uncertainty Analysis Methods

The presentations of each method on Tuesday, 1 March, and the discussion on Wednesday, 2 March, should cover the following matters:

a) Why do we study uncertainties?
What is the licensing status of the method in its own country? Elsewhere?

b) Identification of uncertainties
What are the sources of uncertainty?
What uncertainties are important in modelling LOCAs and other transients?

c) Characterisation of uncertainties
How do we describe uncertainties and why?
Distinguish accuracy, uncertainty, error.

d) Quantification of uncertainties
How are input parameter uncertainties quantified?
How does the method treat code effects including numerical approximation due to the noding scheme?
How is judgement used in quantifying uncertainties?

e) Combination of uncertainties
How does the method combine and propagate uncertainties and why?
How does the method deal with scaling?
How does the method deal with dependencies between uncertain variables?
How are compensating errors dealt with?
How does the method deal with cliff edges?
What part does iteration play in the method?
How is judgement used in combining uncertainties?

f) Applications
At what stage of development is the method?
What studies have used the method so far?
What were the results for the main quantities of interest?

g) Resources (Staff time and computer time)
What resources have been used in developing the method? What resources will be used in further development?
What resources are needed to apply the method to a real problem?
How can this be reduced?

h) Future developments
What developments and applications of the method are planned?

Appendix 3: Proposed International Standard Problem

1 Introduction

The July 1993 meeting of the TGTHSB had proposed an ISP to compare the different methods proposed for uncertainty analysis for thermal-hydraulic codes. The September 1993 meeting of PWG2 called for this proposal to be considered further at the Workshop.

A small group of the Workshop Delegates therefore discussed the plans for the ISP on 4 March 1994, in the light of the discussion on the previous three days.

2 Objectives of the ISP

The meeting reviewed the objectives for the International Standard Problem. It was felt that the objectives presented to the September 1993 meeting of PWG2 were too ambitious. The meeting recommended that they should be revised to be:

1 To gain insights into differences between features of the methods by:

- comparing the different methods, step by step, when applied to the same problem
- comparing the uncertainties predicted for specified output quantities of interest
- comparing the uncertainties predicted with measured values

and so allowing conclusions to be drawn about the suitability of methods.

2 To inform those who will take decisions on conducting uncertainty analyses, for example in the light of licensing requirements.

3 Choice of Experiment

Presentations, proposing candidate experiments, were made.

M. Leonardi, University of Pisa, proposed LSTF SB-CL-18 = ISP26 or LOBI BL-06.

B. Worth, JRC Ispra, proposed LOBI BL-34 or LOBI BL-44.

Other possible candidates identified were: PACTEL (the only possible blind test), OMEGA bundle or tube and Vertical Canon.

A majority (delegates from France, Germany, Italy, and USA (speaking in a personal capacity)) preferred an integral experiment. UK preferred a separate effects test.

After a full discussion of the merits of the various possibilities identified it was agreed to recommend to the TGTHSB that it should choose one of LOBI BL-34 and ISP 26. The Task Group should choose according to the views of the four countries most likely to participate

(France, Germany, Italy and UK). These four countries should inform A. J. Wickett of their views on the two possible tests by the end of April 1994.

The views expressed on the choice of experiment were:

France	1st BL-34	2nd ISP 26	both acceptable
Germany	1st ISP 26		acceptability of BL-34 doubtful
Italy	1st ISP 26	2nd BL-34	both acceptable
UK	1st BL-34	2nd ISP 26	both acceptable.

4 Conduct of the ISP

B. Worth reported that, if a LOBI test were chosen then JRC Ispra would be willing to administer the ISP, if a formal request came from PWG2. A volunteer for this task was not identified, if the LSTF test was chosen.

It was agreed that participants should be encouraged to share information to the greatest extent possible. This might be accomplished by the exchange of monthly reports.

Full relevant data about the test should be provided uniformly to the participants.

There should be a meeting of participants at the stage of ranging input uncertainties.

Participants should be invited to make uncertainty statements and, if they wish, sensitivity statements about specified predicted quantities for example (taken from a slide presented by Leonardi):

rod surface temperature (three levels)
residual mass
primary pressure
relevant time sequence of events.

The choice of input uncertainties would be left to participants.

Final submissions should be made in time for a Workshop to compare the results in April-May 1996.

The submissions should be presented and compared so as to meet the objectives set out above.

Appendix 4: Arguments for and against the use of Subjective Probability Distributions

1. Introduction

Section 5 of this report explained that a majority of the Workshop participants agreed that it was appropriate to use either subjective probability distributions or uncertainty ranges to characterise uncertainties. A minority argued that subjective probability distributions should not be used. It was agreed that the arguments on both sides should be presented in an Appendix to the report.

The arguments in favour of the use of subjective probability distributions, representing the majority view of the Workshop, are presented in Section 2 of the Appendix and were drafted by Horst Glaeser and Eduard Hofer (GRS). The arguments against the use of subjective probability distributions, being the minority view, are presented in Section 3 of the Appendix and were drafted by Tony Wickett (AEA).

2. Arguments for a *Probabilistic* Uncertainty and Sensitivity Analysis

Majority opinion contributed by H. Glaeser and E. Hofer.

The probability distributions in this appendix are called "subjective" since they are to express the states of knowledge of parameter values rather than stochastic variability of random variables. This does not preclude stochastic variability from serving as a basis for state of knowledge quantification wherever appropriate.

Expressing the state of knowledge at the level of parameters and models by subjective probability distributions entails such a distribution for each of the code output quantities. This distribution logically results from the individually quantified states of knowledge and their propagation through the code.

It says "If your states of knowledge are as expressed by the subjective probability distribution at the level of parameters and models then this is the logically resulting state of knowledge at the code output level."

A state of knowledge based on minimum information at the parameter level is expressed in uniform (maximum entropy) distributions. The code does not necessarily (in fact scarcely) translate this into a minimum information distribution at the code output level. This is understandable since the code carries additional information.

Evidence in the form of pertinent observations, experiments, validation experience and theoretical investigations will suggest preferences at the level of parameters and models. The corresponding states of knowledge are expressed by subjective probability distributions other than uniform. They are translated by the code, in a logically consistent manner, into preferences at the code output level. These preferences are important to know.

New evidence may lead to changes in preferences at the parameter level and the analysis will show the logical effect of those changes on the resulting state of knowledge at the code output level.

There will often be state of knowledge dependence between parameters, irrespective of whether the distributions are uniform or not. As a consequence not every combination (or set of combinations) of parameter *values* will make sense or will be feasible as input to the code. The dependence is generally suggested by the underlying physics. There are several convenient ways of quantitatively accounting for state of knowledge dependence in a probabilistic uncertainty and sensitivity analysis. These range from correlation coefficients, to conditional distributions, restrictions imposed by specified inequalities, unknown monotone relationships and the like.

A u% quantile denotes the output value which will not be exceeded with u% subjective probability. Well established concepts and tools from probability calculus and statistics are available to obtain estimates of the quantiles of the resulting code output distributions from a limited number of code runs. Confidence statements in the classical statistical sense can be obtained. They quantify the possible impact of the sampling error which is due to the fact that only a limited number of code runs was performed [1]. The quantiles for code output quantities and their confidence limits are useful for comparisons with deterministic as well as probabilistic safety standards and regulatory limits. The subjective probability for compliance with or violation of a limit is readily obtained. To know, whether there is only a small or substantial probability for violation of a deterministic limit and which of the uncertainties at the level of parameters and models contributes most to this probability is important information.

Sensitivity measures, derived from the sample of parameter and corresponding code output values, rank the uncertain parameters and models with respect to their contribution to the combined effect on code output uncertainty. This provides guidance on where to invest in future improvements of the state of knowledge at the level of parameters and models in order to reduce code output uncertainty most effectively. The sensitivity information may also identify those uncertain parameters and models that seem to be responsible for discrepancies between code output and experimental value.

In conclusion:

- Using only minimum and maximum values of uncertain parameters is most often unsatisfactory as extreme code output values may be observed for intermediate parameter values.

- Trying to find those intermediate parameter values by expert judgement will be successful only for moderately complex relationships in the code. Even then it will be hard, in many practical instances with numerous uncertain parameters and models, unless the number of uncertainties considered is artificially reduced to this end. Reference 2 accounted for 60 uncertain parameters and models. The analysis disclosed maximum values of the most important code output quantities for reasons that were not thought of by the experts prior to the analysis.

- To treat all parameter values between minimum and maximum alike will be wrong if evidence suggests preferences - so preferences need to be processed.

- New evidence may lead not only to changes in preferences but also in minima and maxima at the parameter level. The code output distribution needs to (and will) reflect these changes in a logically consistent manner.

- The code output distributions from a probabilistic uncertainty and sensitivity analysis quantify the combined effect of parameter and model uncertainties.

- The sensitivity measures indicate where the most important contributions to code output uncertainty come from. They account not only for the specific functional relationships in the code but also for the present state of knowledge at the level of parameters and models.

3. Background information and arguments against using subjective probability distributions

Minority opinion contributed by A. J. Wickett.

The conditions under which it may be appropriate to include subjective probabilities in reactor safety analyses have been discussed for a number of years. In the context of LOCA uncertainty analysis, it was recently discussed following publication of the CSAU demonstration. Extracts from that discussion are reproduced in Section 3.1 of this Appendix, as background information, to illustrate the views expressed before the March 1994 Workshop.

A number of statistical textbooks discuss the circumstances under which subjective probabilities can be used. Section 3.2 of this Appendix considers the definition of subjective probability and the arguments used to justify the use of flat distributions to represent minimal knowledge to see whether they can be applied in LOCA uncertainty analysis.

Finally, Section 3.3 reports some further arguments used at the Workshop concerning the place of subjective probability distributions in safety analysis. Section 3.4 summarises the conclusions drawn in Section 3.

3.1. Background information: Discussion of the CSAU Demonstration

The issue of the use of subjective probability distributions was included in the discussion of the CSAU demonstration published in Nuclear Engineering and Design in 1992. The relevant parts of that discussion are reproduced here.

The authors of the CSAU demonstration wrote [3]:

> "It is well known [1] that real experimental data do not follow any of the classical distribution functions, but in order to proceed we must specify an underlying function. We deal with this problem by using the Uniform Distribution. This choice should not be presumed to imply that we believe that the data really are uniform. The uniform distribution requires *only* that the end

points (the range) be specified and does not imply that any value of the parameter is more likely than any other value; it is often referred to as the distribution which maximises lack of knowledge. Under such circumstances the choice of the range is most important. In this study it was made conservatively, and was based on data considerations [4]."

Publication of the CSAU demonstration in Nuclear Engineering and Design was followed by a number of letters to the Editor. These included the following points:

"The seven chosen uncertain parameters are assumed to have flat probability distributions within the prescribed uncertainty bands. This is just one of many possible distributions, and the results could be very sensitive to the shape of the distribution. (For instance the flat distribution assumption for the T_{min} data described above would certainly be non-conservative relative to a distribution based on our understanding of the database)." [5]

"Justification of the choice of a uniform distribution by these words is ridiculous. What a coincidence, that the choice of a uniform distribution gave such 'nice' agreement in subsequent (questionable!) comparisons to data. What are the distributions to model? Stochastic variability with respect to what? Or lack of knowledge about summary values to be used in TRAC?" [6]

In response to these comments, the CSAU authors wrote [7]:

"We preface our response by stating the work was not meant to be an academic exercise in statistical analysis, but rather a solution to a real engineering problem based on experimental data having recognized limitations, and imperfect calculational tools. In our experience, this is often the situation faced by engineers and can only be resolved by performing a *prudent engineering* analysis with the information and tools at hand. We readily admit to a subjective viewpoint in the following response.

The commentors see significant faults because they (variously) believe:
(1) Data scatter was treated as uncertainty.
(2) Uniform distributions were insufficiently justified."

They give the following answers to these points [7]:

"(1) 'Uncertainty' has various meanings. Statistically one can speak of 'uncertainty in the mean' or in various percentiles, 'uncertainty in variation', or many other very specific aspects. More generally in ordinary engineering speech, uncertainty indicates a subjective viewpoint that an answer to a question may not be a single number, but can best be given by a range about some mean value. In this light 'data scatter' most assuredly takes on the aspect of uncertainty. The cause of data scatter may be due to the random realization of various physical phenomena, or to instrument malfunction. However, if data scatter is real, then the answer can only be given in terms of a range and is, thus, uncertainty.

(2) The choice of the uniform distribution is, no matter what the commentors may say, justifiable from a lack of knowledge point of view; but more than that, it requires only the specification of end points. A truncated normal distribution would produce a mean and variance with their own uncertainties. It appeared to us from a subjective viewpoint that the uniform distribution required the least justifications of any postulated distribution. Finally, when truncated normals were used, they produced less conservative, but similar results. These early trials produced results that fit the experimental histograms just as well (from a subjective point of view) as did those obtained with the uniform distribution."

It is clear that the CSAU authors did not fully answer the criticisms of:

- representing uncertainty in a parameter by a single subjective probability distribution and

- the choices of uniform subjective probability distributions.

It is also clear that they did not attempt to set out a rigorous justification of their method. It follows that this discussion did not close the issue.

3.2. Discussion from Statistical Textbooks

One way to help clarify the conditions under which the use of subjective probabilities may be justified is to examine the statistical textbooks that discuss the subject. This section uses information from such textbooks to consider two items: the definition of subjective probability and the use of flat distributions of subjective probability to represent minimal knowledge.

The book by Winkler and Hays defines subjective probability thus [8, Section 2.16]:

"A formal definition of subjective probability can be given in terms of lotteries as follows. Your subjective probability P(A) of an event A is the number P(A) that makes you indifferent between the following two lotteries.

Lottery I: You obtain X with probability P(A)
You obtain Y with probability 1 - P(A)

Lottery II: You obtain X if A occurs
You obtain Y if A does not occur.

Here X and Y are two 'prizes'. The only restriction on X and Y is that one must be preferred to the other; if you are indifferent between X and Y, then you are indifferent between the two lotteries regardless of the choice of P(A). In the above examples, X = $100, Y = $0, and A = 'rain tomorrow'."

To see how this definition works in the context of reactor safety analysis the event A must be replaced by a relevant example, such as: "The pressure drop form loss at the plenum inlet in OMEGA test 9 is between 0.66 and 0.67" (taken from Tables 1 and 2 of Reference 9). This parameter is assessed to be uncertain within the range 0.66 to 1.5. To determine the

analyst's subjective probability he or she must be offered a series of lotteries like I above with different values of P(A) until the analyst is indifferent between I and II.

However this process cannot be used to find P(A) in this case. Since the parameter was not measured the truth of event A cannot ever be determined. So the outcome of Lottery II cannot be determined and it cannot pay out. In these circumstances the rational analyst should always choose Lottery I (which can pay out) over Lottery II (which can never pay out). So there is no value of P(A) for which the analyst is indifferent between I and II.

This problem arises directly from the definition of subjective probabilities. It suggests that they are fundamentally unsuitable for expressing the uncertainty in parameters, that have not been, and will not ever be, measured, such as the one quoted above.

Turning now to the use of flat distributions of subjective probability to represent minimal knowledge, this arises from the use of Bayes' theorem to make inferences from evidence. Bayes' theorem allows a posterior subjective probability distribution for a quantity to be derived from information about the likelihood of obtaining a particular piece of evidence (as a function of the actual (unknown) value of the quantity) and a prior subjective probability distribution, defined before the evidence was known. The posterior probability distribution includes the information from the evidence. It is subjective because the prior distribution was subjective.

Books such as Box and Tiao carefully explain the conditions under which this process can be used [10]. Section 1.3 of reference 10 describes the selection of "noninformative priors". Under certain circumstances, there defined, such a prior distribution can be a uniform one. In choosing a non-informative prior,

> "the main issue is how to select a prior which provides little information relative to what is expected to be provided by the intended experiment."

Thus a non-informative prior is required, for example, when measurements of a quantity that has not been measured before are to be analyzed. Such a distribution is not, therefore, claimed to be non-informative in any absolute sense, but only when compared with information to be provided from measurements. The use of such a distribution is only justified when it is to be combined with such information, and not otherwise.

3.3 Further arguments against using subjective probability distributions

It was argued at the Workshop that assigning subjective probability distributions adds information and hence leads to the understatement of uncertainty. For example, if complete ignorance of a single valued quantity, that has not been measured, within a range is represented by a flat distribution this representation will lead to the same conclusions as the knowledge that the quantity is uniformly (and in no other way) distributed.

The proposition that a flat distribution of subjective probability uniquely represents the minimum knowledge can be disproved by considering what happens if the independent variable, x, is transformed to another variable, say $y = x^2$. Minimum knowledge cannot be represented by flat distributions in both x and y at the same time.

Finally it was argued that subjective probability distributions would change, as a result of new evidence, in a way that was undesirable in a safety case. An addition of new relevant information should lead to a reduction in uncertainty in the output quantity of interest. If new evidence is produced that is inconsistent with part of an uncertainty range, then an analyst will presumably reduce his or her subjective probability density to (or nearly to) zero in that part of the range. It will therefore have to increase in all or part of the rest of the range. An increased subjective probability density of an input parameter, in all or part of its range, in an uncertainty study may increase the subjective probability density of the output variable in part of its range. This may result in the range between the 5th and 95th percentiles of the distribution being extended in either direction, so introducing values of the output quantity into the 5-95% range that were outside it before the new evidence was produced. Thus possibilities, previously ruled out on a 95% basis, now cannot be ruled out on this basis. This is illustrated in Figure 1.

If the uncertainty range given for a quantity had the property that new evidence could shift it upwards (or downwards) so as to include new values that had not been included before, such a range could not be used as a reliable basis for decisions. It is therefore undesirable for uncertainty statements for quantities, that are to be used in a safety case, to be able to change in this way. Licensing bodies, who accept uncertainty statements based on subjective probability distributions, should be aware that they have this property.

3.4 Conclusions

- Discussion of the CSAU demonstration in Nuclear Engineering and Design did not resolve criticisms of the use of single subjective probability distributions and the selection of uniform distributions.

- The textbook definition of subjective probabilities - in terms of lotteries - does not support the use of subjective probabilities to express the uncertainty of parameters that have not, and never will be, measured.

- The use of a uniform subjective probability distribution can only be described as "non-informative" in an analysis which will combine the distribution with evidence which will dominate the result.

- Use of a single subjective probability distribution for an uncertain parameter whose value is unknown within a range is equivalent to adding information.

- A uniform distribution of subjective probability within a range cannot uniquely represent minimum knowledge.

- The introduction of new evidence could cause a 5-95% subjective probability range for a calculated quantity to extend so as to take in some values previously outside the range. This property is undesirable for predictions in a safety case, and licensing bodies should be aware that uncertainty statements based on subjective probability distributions have this property.

4. References for Appendix 4

1 W. J. Conover, "Practical Nonparametric Statistics", 2nd Edition (Wiley, New York, 1980)

2 H. Glaeser et al, "Uncertainty and Sensitivity Analysis of a Post-Experiment Calculation in Thermalhydraulics", Reliability Engineering and System Safety, 44 (1994)

3 G. S. Lellouche *et al,* "Quantifying Reactor Safety Margins Part 4: Uncertainty Evaluation of LBLOCA Analysis based on TRAC-PF1/MOD1", Nuclear Engineering and Design, 119, 67-95 (1990)

4 W. Wulff *et al,* "Quantifying Reactor Safety Margins Part 3: Assessment and Ranging of Parameters", Nuclear Engineering and Design 119, 33-65 (1990)

5 A. J. Wickett and I. Brittain, "Quantifying reactor safety margins: What remains to be done", Nuclear Engineering and Design 132, 405-408 (1992)

6 E. Hofer *et al,* "Comments on 'Quantifying Reactor Safety Margins'", Nuclear Engineering and Design 132, 411-414 (1992)

7 G. E. Wilson *et al,* "TPG response to the foregoing letters-to-the-editor", Nuclear Engineering and Design 132, 431-436 (1992)

8 R. L. Winkler and W. L. Hays, "Statistics: Probability, Inference and Decision", second edition, Holt, Rinehart and Winston, New York (1975)

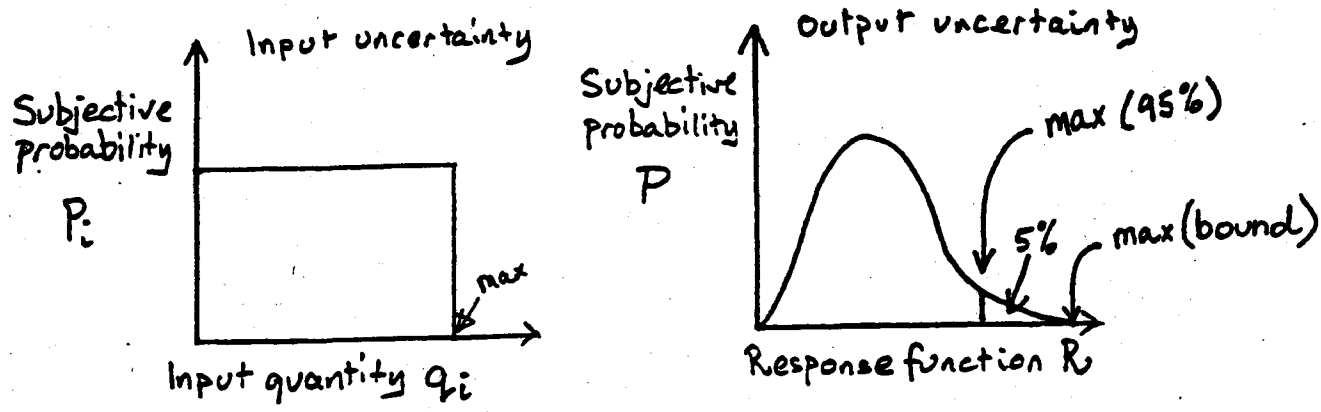
9 H. Glaeser *et al,* "Uncertainty and Sensitivity Analysis of a Post-Experiment Calculation in Thermal Hydraulics", Volume 2 of this report.

10 G. P. Box and G. C. Tiao, "Bayesian Inference in Statistical Analysis", Addison - Wesley, Reading (1973)

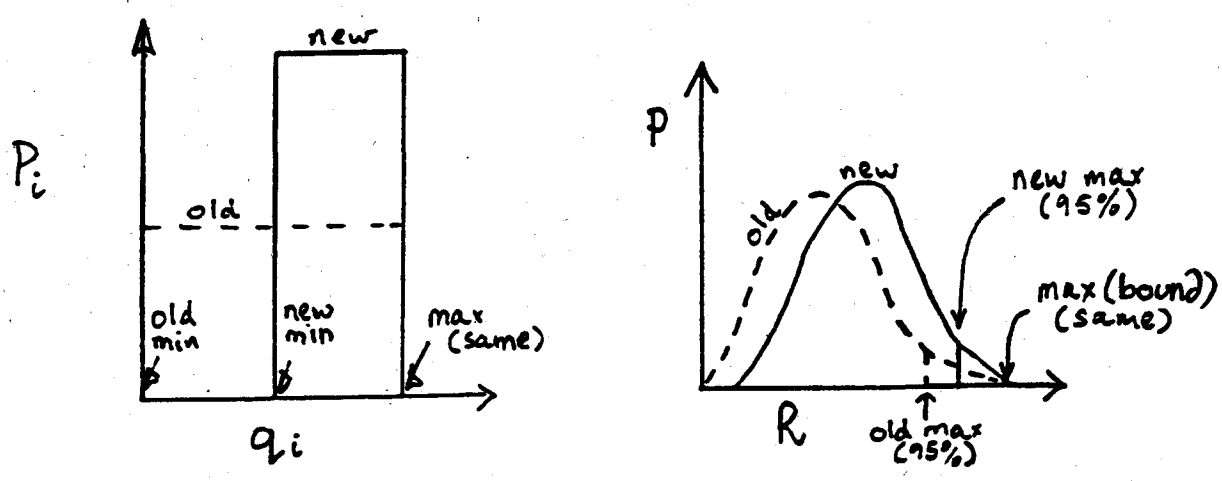
Initial evidence. Uncertain input quantity q_i is uncertain over a range and the analyst assigns a flat subjective probability distribution, $p_i(q_i)$. The output quantity of interest (response function), R, is a function of several uncertain input parameters including q_i,

$$R = R(q_1, \ldots q_i, \ldots).$$

The analyst's subjective probability distribution function for R, $P(R)$, is calculated from the subjective probability distributions assigned to the input parameters. The 95th percentile of $P(R)$ is presented in a safety case.



New evidence. New evidence now restricts the range of input quantity q_i so that the range minimum is increased and the range maximum remains unchanged. In this particular example, R is assumed to be an increasing function of q_i.



The 95th percentile of $P(R)$ has now increased because new evidence has ruled out low values of q_i and R. Safety case must be modified by presenting an increased 95th percentile of $P(R)$.

Figure 1. Possible effect of new evidence on 95th percentile of response function calculated using subjective probability distribution functions.